HANDBOOK FOR MATHEMATICS GRADUATE STUDENTS

University of Kentucky

Department of Mathematics

Director of Graduate Studies
Department of Mathematics
University of Kentucky
Lexington, KY 40506-0027

859 257 6808 [Graduate Secretary]
800 357 1113 [toll-free]
859 257 4078 [FAX]

math-dgs@ms.uky.edu

June 2004

Table of Contents

Mathematics at Kentucky

The Department of Mathematics offers programs leading to the degrees of Master of Arts, Master of Science, and Doctor of Philosophy. The full-time faculty has won substantial national and international recognition. In recent years, faculty have been invited to international institutes such as Oberwolfach, the Max-Planck Institut, the Mathematical Sciences Research Institute and the Institute for Mathematics and its Applications. Many faculty have won research grants (including an NSF Career award) from the National Science Foundation and other agencies. Two faculty members have held Centennial Fellowships from the AMS in recent years.

The department maintains a colloquium series as well as many weekly research seminars. The colloquium series brings distinguished visitors to the campus. In 2000 we initiated the Hayden Howard lecture series which brings a distinguished mathematician to campus for a lecture. In the fall of 2001, the van Winter Lecture series in mathematical physics was inaugurated with a lecture by Elliot Lieb of Princeton University.

Offices and a library for the mathematical sciences are located in a modern, air-conditioned office building. The library contains a comprehensive collection of books and journals in mathematics, statistics and related areas. The Center for Computational Science provides computing facilities for faculty and students including a Hewlett-Packard Superdome computer and a Linux cluster. There are also numerous public computer labs available around the campus. In addition, HP workstations, a Linux cluster and numerous desktop computers are available to the mathematical sciences departments. The Center for Computational Science also supports graduate student research through fellowship awards. The Department is a member of the Institute for Mathematics and its Applications (IMA) in Minneapolis, Minnesota. Through this membership, graduate students may participate in several summer programs for graduate students. In addition, our membership provides funds for graduate students to attend programs at the Institute and conferences at other institutions affiliated with the IMA.

Financial support for graduate students is available in the form of Teaching Assistantships, Fellowships and Research Assistantships; these are awarded on a competitive basis.

The Faculty and Their Research

Algebra

- James Beidleman, Professor, Ph.D., Pennsylvania State, 1964, Group Theory.
- Donald B. Coleman, Professor Emeritus, Ph.D., Purdue, 1961, Algebra.
- Alberto Corso, Assistant Professor, Ph.D., Rutgers University, 1995, Commutative Algebra.
- Paul M. Eakin, Jr., Professor, Ph.D., Louisiana State, 1968, Commutative Algebra, Mathematics Outreach.
- Edgar Enochs, Professor, Ph.D., Notre Dame, 1958, Module Theory, Homological Algebra.
- Kenneth Kubota, Professor, Ph.D., Faculté des Sciences de Paris, 1969, Computer Operating systems and Number Theory.
- David Leep, Professor, Ph.D., Michigan, 1980, Algebra and Number Theory.
- Uwe Nagel, Assistant Professor, Ph.D., University of Paderborn, 1990, Algebraic Geometry.
- Avinash Sathaye, Professor, Ph.D., Purdue, 1973, Algebraic Geometry.

Analysis and Partial Differential Equations

- David Adams, Professor, Ph.D., University of Minnesota, 1969, Potential Theory.
- James Brennan, Professor, Ph.D., Brown University, 1969, Complex Analysis.
- Russell Brown, Professor, Ph.D., University of Minnesota, 1987, Harmonic Analysis and Partial Differential Equations.
- J.D. Buckholtz, Professor Emeritus, Ph.D., University of Texas, 1960, Complex Analysis.
- Richard Carey, Professor, Ph.D., SUNY/Stony Brook, 1970, Functional Analysis.
- Raymond H. Cox, Professor Emeritus, Ph.D., University of North Carolina, 1963, Functional Analysis.
- Michael Freeman, Professor Emeritus, Ph.D., University of California at Berkeley, 1965, Several Complex Variables.
- Ronald Gariepy, Professor, Ph.D., Wayne State University, 1969, Partial Differential Equations.

- Lawrence A. Harris, Professor, Ph.D., Cornell University, 1969, Infinite Dimensional Holomorphy.
- Peter Hislop, Professor, Ph.D., University of California at Berkeley, 1984, Mathematical Physics and Riemannian Geometry.
- Henry Howard, Professor Emeritus, Ph.D., Carnegie Institute of Technology, 1958, Ordinary Differential Equations.
- Michel Jabbour, Assistant Professor, Ph.D., 1999, California Institute of Technology, Mathematics of Materials.
- John Lewis, Professor, Ph.D., University of Illinois, 1970, Complex Analysis, Harmonic Analysis and Partial Differential Equations.
- Chi-Sing Man, Professor, Ph.D., Johns Hopkins University, 1980, Continuum Mechanics, Mathematics of Materials.
- Richard Millman, Professor Ph.D., Cornell University, 1971, Mathematics Outreach, Differential Geometry.
- Robert Molzon, Associate Professor, Ph.D., Johns Hopkins University, 1977, Several Complex Variables.
- Peter Perry, Professor and Chair, Ph.D., Princeton University, 1981, Mathematical Physics and Riemannian Geometry.
- Wimberly Royster, Professor Emeritus, Ph.D., University of Kentucky, 1952, Complex Analysis.
- Raymond Rishel, Professor Emeritus, Ph.D., University of Wisconsin, 1959, Stochastic Control Theory.
- Zhongwei Shen, Professor, Ph.D., University of Chicago, 1989, Partial Differential Equations and Harmonic Analysis.
- Ted Suffridge, Professor, Ph.D., Kansas, 1965, Complex Analysis.
- Changyou Wang, Associate Professor, Ph.D., Rice University, 1996, Non-linear Partial Differential Equations.

Discrete mathematics

- Richard Ehrenborg, Associate Professor, Ph.D., Massachusetts Institute of Technology, 1993, Algebraic combinatorics.
- Carl Lee, Professor, Ph.D., Cornell University, 1981, Polytopes, Combinatorics and Mathematics Outreach.
- Margaret Readdy, Associate Professor, Ph.D., Michigan State University, 1993, Algebraic combinatorics.

Numerical Analysis and Scientific Computation

- Thomas L. Hayden, Professor, Ph.D., Texas, 1961, Matrix Minimization Problems.
- Sung Ha Kang, Assistant Professor, Ph.D. 2002, UCLA, Image Processing.
- Seongjai Kim, Assistant Professor, Ph.D. Purdue University, 1995, Numerical Solution of Partial Differential Equations and Geophysical Problems.
- Ren-Cang Li, Associate Professor, Ph.D. University of California at Berkeley, 1995, Numerical Linear Algebra.
- James H. Wells, Professor Emeritus, Ph.D., Texas, 1958, Functional Analysis.
- Qiang Ye, Associate Professor, Ph.D., University of Calgary, 1989, Numerical Linear Algebra.

Topology

- Marian Anton, Assistant Professor, Ph.D. University of Notre Dame, 1998, Algebraic Topology.
- Thomas A. Chapman, Professor, Ph.D., Louisiana State University, 1970, Geometric Topology.
- Carl Eberhart, Professor, Ph.D., Louisiana State University, 1966, Point Set Topology.
- Brauch Fugate, Professor, Ph.D., University of Iowa, 1964, General Topology.
- Vassily Gorbounov, Associate Professor, Ph.D., 1986, Novosibirsk University, Homotopy Theory.
- David Johnson, Professor Emeritus, Ph.D., University of Virginia, 1970, Homotopy Theory.
- John Mack, Professor Emeritus, Ph.D., Purdue, 1959, General Topology.
- Serge Ochanine, Associate Professor, Ph.D., Université Paris XI (Orsay), 1978, Homotopy Theory, Elliptic Cohomology.

Master's Degree Programs

M.A. and M.S. in Mathematics

The Department offers three Masters degrees, M.A. and M.S. in Mathematics and M.S. in Applied Mathematics. The M.A. and M.S. in Mathematics are 30 hour Master's degrees offered under Plan A (thesis option) or Plan B (non-thesis option). Most students take the non-thesis option and these requirements are detailed here. Students interested in writing a Master's thesis should consult with the Director of Graduate Studies and the Graduate School Bulletin for the requirements. To receive a M.S or M.A. in Mathematics, students must complete course work and a final examination.

The coursework requirements are:

- Complete 30 hours of graduate work in Mathematics and related areas with a final gradepoint of 3.0 or better.
- At least 20 hours in Mathematics (MA and cross-listed courses).
- At least 15 hours at the 600 level and above.
- At least 12 hours in Mathematics and at the 600 level and above.

All students in the Masters program must pass a final exam. This requirement is typically met by having the student read a paper or part of a monograph and give a presentation of this material to the examination committee. The presentation is prepared in consultation with the chair of the committee. The goal of the examination is to evaluate whether the student can read and present mathematics.

Subject to the approval of the Director of Graduate Studies, the candidate may submit as part of the required course work a maximum of nine credits of transfer work and courses taken outside the Department.

The M.A. is not a one year program, although some students do manage to complete it in two semesters and a summer. Typically, a student chooses Plan B and takes three semesters or stays on for two academic years, taking one semester of more advanced course work. There is great flexibility in the courses a student may take for the M.A. degree. Most students in the Ph.D program will automatically satisfy the requirements for the M.A under Plan B.

Recommendations for Applied Mathematics (M.S. in Mathematics.) These recommendations are designed to prepare students for industrial, management or public service employment. They emphasize the skills, attitudes, and knowledge needed for recognition, formulation and solution of real-world problems. Each student should complete 24 credits from the following courses:

- Real and Complex Analysis (MA 575, MA 671, MA 676);
- Numerical Analysis (MA 522, MA 537, MA 625,),
- Optimization (MA 515, MA 618)
- Matrix Theory and Linear Algebra (MA 522, MA 622)
- Partial Differential Equations (MA 533, MA 633).

Students should also complete 6 credits in Computer Science or Statistics, such as:

- Statistics and Probability (STA 524, STA 525, STA 624);
- Computer Science (CS 420G, CS 505, CS 545).

Students who desire a more intensive program should consider the M.S. in Applied Mathematics.

Master of Science in Applied Mathematics

The Master of Science in Applied Mathematics is intended for students interested in pursuing a career in business, industry or government service. The program should appeal to students who have just finished undergraduate degrees as well as individuals who currently hold positions that may involve mathematical applications. This program is more intensive than the Master of Science in mathematics program described above. In particular, the program requires 36 hours of graduate work. Students who enter the program will be expected to have basic undergraduate mathematical training. The course requirements for a typical engineering degree will, in most cases, include sufficient mathematical material to allow a student to proceed directly to the required and recommended course work. Specific options or areas of concentration within the program include:

- Computational Fluid Dynamics
- Numerical Analysis and Computation
- Operations research, Optimization, and Financial Mathematics
- Materials Science.

In addition, students may design their own option to satisfy specific interests. See the Director of Graduate Studies or the Director of the Applied Master's program for more information. Upon admission to this program, a student will be assigned an advisor who will assist the student in selecting courses that best match the student's area of concentration. Thirty six (36) total credit hours are required for graduation from the program. Specifically the course requirements are:

- Fifteen (15) hours in Mathematics including MA 565 and MA 575,
- Three (3) hours of a mathematical problem seminar,
- Twelve (12) hours in Statistics or Computer Science,
- Six (6) hours in an outside minor,
- At least eighteen (18) hours at the 600 level or above,
- At least fourteen (14) of the hours in the first three categories must be at the 600 level or above.

Since industry problems almost always require computational implementation of mathematical models, all students must be competent in computer programming. Normally, a working knowledge of C++, or Fortran will be required; however, knowledge of other languages as alternatives that meet the requirements of the area of concentration may be approved by the student's advisor. Competence in programming will be expected in the required and elective course work.

All students in the Master's of Science in Applied Mathematics program must pass a final exam. This requirement is typically met by having the student read a paper or part of a monograph and give a presentation of this material to the examination committee. The presentation is prepared in consultation with the chair of the committee. The goal of the examination is to evaluate whether the student can read and present mathematics. The presentation may be based on work completed in the problem seminar.

The Doctoral Program in Mathematics (Ph.D.)

The Doctoral Program in Mathematics is a program of study and research which has five components: (1) breadth component; (2) foreign language component; (3) minor field component; (4) research component; and (5) residence component. Each of these components is described below.

Breadth Component

The graduate program in mathematics is divided into six basic areas: (1) Algebra; (2) Analysis; (3) Topology; (4) Differential Equations (ordinary or partial); (5) Numerical Analysis; and (6) Discrete Mathematics. In the first two years of graduate study, the students are expected to pursue a broad course of study and prepare themselves to pass written Preliminary Examinations in three of these six areas and an oral Qualifying Examination in their area of specialization. The details of these examinations follow in the section on Ph.D. Examinations.

Foreign Language Component:

Students should satisfy the Foreign Language Requirement described under “General Requirements for all Master's Degrees” in the Graduate School Bulletin (pg. 20 of 2000-2002 edition).

Acceptable languages are ordinarily Chinese, French, German, and Russian. On the recommendation of the Advisory Committee, the Director of Graduate Studies may allow another language to be used. Such substitutions will be made only if the student can demonstrate the existence of a substantial body of mathematical research, in the student's major area, which is written in the requested language.

The Foreign Language Component may be completed before or after the Preliminary Examinations. However, it must be completed before taking the Qualifying Examination.

Minor field component:

The minor requirement for the Ph.D. program may be satisfied in one of the following three ways.

1. Complete a preliminary examination sequence and two additional graduate courses in an area of mathematics distinct from the area of the student's planned dissertation.

2. Complete (take the courses or pass the examination) four preliminary examination sequences plus one additional graduate course in one of these four preliminary examination areas. The additional course must be taken in an area that is distinct from the planned dissertation.

3. Complete a two course sequence at the graduate level in a department outside of mathematics, but related to mathematics. The courses should have the prior approval of the DGS or advisory committee.

Research Component:

After having passed the Qualifying Examination, each student shall pursue a course of study leading to the writing of a doctoral dissertation, which is expected to be a significant piece of original mathematical research. This dissertation must be of sufficient quality and depth to satisfy the graduate mathematics faculty and the student's dissertation advisor. It is expected that the research will be acceptable for publication in an appropriate scholarly journal and that the student will submit the results of the dissertation for publication.

Residence Component:

A minimum of three academic years of full-residence graduate work is required for the doctorate; at least one year of residence is required after the Qualifying Examination and before the degree is conferred. See the Graduate School Bulletin for details.

Preliminary and Qualifying Examinations

1. Students will take three Preliminary Examinations and one Qualifying Examination.
2. Preliminary Examinations: each of the three Preliminary Examinations will be a written three hour test and will cover two semesters of course work, at the 500 level or above, in one of the following areas: Analysis, Topology, Algebra, Partial Differential Equations, Discrete Mathematics, and Numerical Analysis.

- Every year, Preliminary Examinations are given prior to the start of the spring semester and the eight week summer session. A student may take one, two or three examinations at those times. Students must pass three Preliminary Examinations within three calendar years of entering the mathematics graduate program. Students who do not pass three examinations in three years may be dismissed from the mathematics graduate program. Except for the three year time limit, there is no limit on the number of times a student may take an examination. Students are strongly encouraged to attempt a Preliminary Examination as soon as they have completed the appropriate courses for that examination.
- Qualifying Examination: With the guidance and approval of the student's Advisory Committee, the student will choose a topic in the area of

proposed research. The student then prepares an oral presentation on this topic. Following a 30 to 45 minute lecture by the student, the Advisory Committee will examine the student over the topic. The lecture and questioning together should not exceed two hours. The Qualifying Examination cannot be taken until the student has passed all three Preliminary Examinations and fully satisfied the Foreign Language Requirement. On successfully completing the Qualifying Examination, the student becomes a Ph.D. Candidate. The Qualifying Examination must be passed within three and a half years of entering the graduate program.

Preliminary Examination in Algebra

Basic Courses: MA 561, MA 661.

Suggested texts: *Abstract Algebra, 2nd edition*, Dummit and Foote,
*Algebra***,** *3rd edition***,** Serge Lang.
*Algebra***,** Thomas Hungerford.

Outline:

- Linear Algebra (not usually covered explicitly in the course): Basic definitions, dimension, matrices and linear transformations, eigenvectors and eigenvalues.
- Groups: Basic definitions, isomorphism theorems, permutation groups, structure of finitely generated abelian groups, groups acting on sets, the Sylow theorems, solvable groups. Dummit and Foote Chapters 1-5, Lang, Chapter 1, sections 1-6 and 8, Hungerford, Chapters 1 (sections 2-6), 2 (sections 1, 2, 4-8).
- Rings: Basic definitions, prime ideals, Euclidean rings, principal ideal domains and unique factorization domains, localization, polynomial rings and power series rings. Dummit and Foote Chapters 7-9, Lang, Chapters 2, 4 (sections 1-3), Hungerford, Chapter 3, 8 (sections 1-3)
- Modules: Basic definitions, vector spaces, modules over a principal ideal domain. Dummit and Foote, Chapters 10-12, Lang, Chapter 3 (sections 1, 2, 5, 7), 15 (section 2) Hungerford, Chapter 4 (sections 1, 2 and 6)
- Fields: Algebraic extensions, separable and purely inseparable extensions, splitting fields, Galois theory, finite fields, symmetric polynomials. Dummit and Foote, Chapters 13, 14, Lang, Chapters 5, 6 (sections 1-3 and 7) Hungerford, Chapter 5, (sections 1-9).

Revised August 2002.

Preliminary Examination in Analysis

Basic Courses: MA 575, MA 676, and MA 671. Students choose to be examined on either i) Real Analysis, MA 575 and MA 676 or ii) Complex Analysis, MA 575 and MA 671. However, students taking the analysis examination must complete all three courses,

Advanced Calculus

***Suggested Texts*:** *Principles of Mathematical Analysis,* W. Rudin, McGraw-Hill, Chapter 1-7.

Mathematical Analysis, T.M. Apostol, Addison Wesley, chapters 1-5, 8, 9, 12, 13 (cf. also Theorem 15-23).

Advanced Calculus, R. C. Buck, McGraw-Hill, chapter 1, 2, section 3.3, 4.1-4.3.

Outline:

- The field of complex numbers $\mathbf{C}$; the real number system $\mathbf{R}$ as an ordered field; the extended real number system.
- Basic topology: finite, countable, and uncountable sets, metric spaces with an emphasis on $\mathbf{R}^n$, compactness and connectedness, Heine-Borel and Bolzano-Weierstrass theorems.
- Sequences and series: convergence criteria, Cauchy sequences, convergence and completeness, manipulations with series.
- Continuity: Limits and continuity for real-valued functions with an emphasis on functions $f:U \to \mathbf{R}^m$ where $U \subset \mathbf{R}^n$. Intermediate Value Theorem.
- Differentiation of functions of one variable: mean value theorems, Taylor's theorem, l'Hopital's rule, differentiation of vector-valued functions.
- Integration of functions of one variable: definition, existence and properties of the Riemann-Stieltjes integral, integration of vector-valued functions.
- Sequences and series of functions: uniform convergence of series and continuity, integration and differentiation of functions defined as series. Equicontinuous families, the Arzela-Ascoli Theorem, and the Stone-Weierstrass Theorem for continuous functions on a finite interval.

Real Analysis

Suggested texts:

Chapter 1-5, 7 of *Measure and Integral*, R.L. Wheeden and A. Zygmund, Marcel Dekker (1977).

Real Analysis, H.L. Royden, Macmillan (1968).

Real and Complex Analysis, W. Rudin, McGraw-Hill.

Outline:

- Lebesgue measure on the real line including outer measure, measurable sets, measurable functions, Cantor ternary set, Cantor ternary function.
- The Lebesgue integral, measurable subsets of the reals, convergence theorems and their relation to integration, the relation between Lebesgue integration and Riemann integration, examples and counterexamples that illustrate the above.
- Differentiation and integration of monotone functions, functions of bounded variation, absolute continuity and differentiation of an integral.

Complex Analysis

Suggested texts:

Complex Analysis, L. Ahlfors, McGraw-Hill (1979) 3rd edition.

Real and Complex Analysis, W. Rudin, McGraw-Hill.

Functions of One Complex Variable, John Conway, Springer-Verlag.

Outline:

- Analytic Functions and Equivalent Conditions; e.g. limit of difference quotient, CR equations, Morera theorem, Power series.
- Cauchy's theorem, Cauchy integral formula, Cauchy's estimates and consequences; e.g., Liouville's theorem, open mapping property, theory of residues.
- Definition and mapping properties of elementary functions; e.g., exponential, trigonometric, and rational functions, special properties of linear fractional transformations.
- Power series, region of convergence, boundary behavior.
- Singularities; removable singularities, poles, essential singularities, branch points.
- Convergence of sequences of functions, normal families.

Preliminary Examination in Discrete Mathematics

The preliminary exam in optimization will change to an examination in discrete mathematics beginning with the June 2003 examinations. The following description is for students who have completed the sequence MA515 and MA618 by the spring of 2002. The syllabus for the new sequence in discrete mathematics appears below.

Basic courses: MA515, MA618

Suggested Texts: *Introduction to Linear Optimization*, D. Bertsimas and J.N. Tsitsiklis, Athena Scientific, 1997, ISBN 1-886529-19-1. [Chapters 1-5, 6 (omitting 6.5), 7 (omitting 7.7 and 7.8), 8, 10, 11.1-11.3].

Foundations of Combinatorial Optimization, J. Lee, manuscript.

Combinatorial Optimization: Theory and Algorithms, B. Korte, J. Vygen, Algorithms and Combinatorics 21, Springer, 2000, ISBN 3-540-67226-5. [Chapters 1-3, 6-10 (omitting 6.4, 7.3, 8.4-8.7, 9.3, 9.5,10.4), 13 (omitting 13.7)].

Additional References: *Linear Programming*, V. Chvatal.

Linear Programming Lecture Notes, C. Lee.

Applications and Algorithms, W.L. Winston.

Integer Programming - Integer and Combinatorial Optimization, G.L. Nemhauser and L.A. Wolsey.

Combinatorial Optimization: Networks and Matroids, E.L. Lawler.

Outline:

- **Linear Programming:**
  - Linear inequalities (Fourier-Motzkin elimination, theorems of the alternative, polyhedral geometry).
  - Simplex methods (primal, dual, parametric primal-dual).
  - Sensitivity analysis.
  - Degeneracy (cycling and its resolution).
  - Revised simplex method.
  - Duality and complementary slackness.
  - Dantzig-Wolfe decomposition.
  - Knapsack problem (intro. to dynamic and integer programming).
  - Column generation (the cutting stock problem).
  - Total unimodularity.
  - Network flows and the network simplex method.
  - Subgradient optimization.
  - Ellipsoid method.

- **Combinatorial Optimization:**
  - Greedy algorithms for spanning trees and matroids.
  - Prim's spanning tree method.
  - Shortest path algorithms.
  - Maximum-weight matroid intersection (including bipartite matching).
  - Max-flow/min cut; Edmonds-Karp specialization of the Ford-Fulkerson algorithm.
  - Nonbipartite matching theory and applications.
  - Formulating integer programs.
  - Gomory cutting planes and Chvatal-Gomory rounding.
  - Branch and bound.
  - Polyhedra and integer programs.
  - Combinatorial consequences of the ellipsoid method.
  - Submodularity and matroid polyhedra.
  - Integral polyhedra.

Preliminary Examination in Discrete Mathematics

The new exam in discrete mathematics will be given for the first time in June 2003.

Courses:

MA 515 and MA 614

Outline:

Linear Programming and Combinatorial Optimization

- Linear inequalities and polyhedra.
- Simplex methods (primal, dual).
- Revised simplex method.
- Duality and complementary slackness.
- Spanning trees.
- Shortest path algorithms.
- Max-flow/min cut; Edmonds-Karp specialization of the Ford-Fulkerson algorithm; Menger's Theorem.
- Matching theory and applications.
- Matroids; Greedy algorithm; Matroid polytope; Maximum cardinality matroid intersection.

Enumerative combinatorics

- Generating functions.
- Stirling numbers of the first and second kind.
- Permutations and permutation statistics.
- q-analogues.
- The twelvefold way.
- Principle of inclusion-exclusion.
- Partially ordered sets and lattices.
- The fundamental theorem of distributive lattices.
- The incidence algebra.
- The Möbius inversion formula.
- The Möbius function and computational techniques.
- The Möbius algebra.
- Semi-modular lattices and hyperplane arrangements.
- The zeta polynomial.
- Rank-selection.

- *R*-labelings.
- Eulerian posets.
- Exponential generating functions.
- The exponential formula.
- Tree enumeration.
- Lagrange inversion formula.

Suggested Texts

B. Korte and J. Vygen, *Combinatorial Optimization: Theory and Algorithms*, Algorithms and Combinatorics 21, Springer, 2002. ISBN 3-540-43154-3.Chapters 1--3, 6--10 (omitting 6.4, 7.3, 8.4--8.7, 9.3, 9.5, 10.4), 13 (omitting 13.7).

D. Bertsimas and J. N. Tsitsiklis, *Introduction to Linear Optimization*, Athena Scientific, 1997. Chapters 2-4.]

C. Lee, *Linear Programming Lecture Notes*, manuscript

R. P. Stanley, *Enumerative Combinatorics*, Vol. 1, Cambridge Studies in Advanced Mathematics, 49, Cambridge University Press, 1997. Chapters 1, 3 and 5.

Additional References:

V. Chvátal, *Linear Programming*, Freeman and Co., 1983.

W. L. Winston, Operations Research: Applications and Algorithms, Duxbury Press, 1987.ISBN 0-87150-065-5 90-01.

G. L. Nemhauser and L. A. Wolsey, *Integer and Combinatorial Optimization*, Wiley, 1999.ISBN 0-471-35943-2.

E. L. Lawler, *Combinatorial Optimization: Networks and Matroids*, Holt, Rinehart and Winston, 1976.

R. P. Stanley, *Enumerative Combinatorics*, Vol. 2, Cambridge Studies in Advanced Mathematics, 62, Cambridge University Press, 1999.

J. H. van Lint and R. M. Wilson, *A Course in Combinatorics*, second edition, Cambridge University Press, 2001.

H. S. Wilf, *Generatingfunctionology*, second edition, Academic Press, Inc., 1994.

MA 515, 618 updated August 2002.
MA 515, 614 updated October 2002.

Preliminary Examination in Numerical Analysis

The old preliminary examination in numerical analysis covers numerical linear algebra and numerical methods for partial differential equations. This exam is to be taken by students who completed the prelim sequence by December of 2002. The new preliminary examination sequence covers MA 537 and MA 522.

Basic courses: MA522, MA625 (old style) MA 522, MA 537 (new style)

Numerical Linear Algebra-MA 522

Suggested text: *Applied Numerical Linear Algebra,* J. Demmel, SIAM, Philadelphia, 1997.

*Matrix Computations, 2nd ed***,** Golub and van Loan, The Johns Hopkins Univ. Press, 1989.

Outline: (and corresponding chapters/sections in Golub and van Loan):

- Basic Matrix Analysis: norms, orthogonality, projection, finite precision. (Ref: Demmel 1.3-1.7 or Golub-van Loan 2.1-2.5)
- Linear Systems of Equations: Gaussian elimination and its variations, error analysis, perturbation analysis, iterative refinement, special linear systems: symmetric, band, block tridiagonal matrices. (Ref: Demmel 2.2-2.5, 2.7, Golub-van Loan 3.1-3.5, 4.1-4.5)
- Linear Least Squares Problem and Orthogonalization: Gram-Schmidt orthogonalization, QR factorization. The full rank and rank deficient least squares problems, perturbation theory. (Ref: Demmel 3.2-3.6, Golub-van Loan 5.1-5.5)
- Eigenproblems and Singular Value Decomposition: Properties and decompositions, QR-like numerical algorithms, perturbation theory. (Ref: Demmel 4.2-4.4, 5.2-5.4Golub-van Loan 7.1-7.5, 8.1-8.4)

Numerical Methods for Differential Equations - MA625

Suggested Text:

Numerical Solution of Partial Differential Equations. K.W. Morton & D.F. Mayers, Cambridge University Press.

References: *Numerical Solutions of Partial Differential Equations by the Finite Element Method*. C. Johnson, Cambridge University Press.

Numerical Methods for Differential Equations. S. Kim, Lecture Notes www.ms.uky.edu/~skim/LectureNotes/NumerPDE.ps.

Outline:

- Parabolic Equations (§ 2.10-15 & Ch. 3)
  - The ?-method
  - Maximum principle and convergence
  - Heat conservation
  - Locally one-dimensional (LOD) method
- Hyperbolic Equations (§4.2-7)
  - The CFL condition
  - Fourier analysis
  - Numerical schemes for conservation laws
- Properties of Numerical Schemes (§5.2-6)
  - Finite difference method
  - Consistency
  - Convergence and stability
  - Accuracy
- Elliptic Equations (§6.4-7)
  - Error analysis and maximum principle
  - Boundary conditions
  - Variational formulation and finite element method

Numerical Analysis - MA537

Suggested Texts:

Numerical Analysis, Mathematics of Scientific Computing, 3rd Edition, D. Kincaid and W. Cheney, Brooks/Cole, 2002. (Sections 3.1-3.4, Sections 6.1-6.4, 6.8, 6.9, Sections 7.1-7.4, Sections 8.1-8.9.)

Numerical Methods using MATLAB. J.H. Mathews & K.D. Fink, Prentice Hall. (Chapter 2, Chapter 4, Chapter 6, Chapter 7, Chapter 9.)

References:

Introduction to Numerical Analysis, J. Stoer & R. Bulirsch, Springer-Verlag

Topics:

- The Solution of Nonlinear Equations:
    - Fixed-point iterations, Newton's method
- Interpolation and Polynomial Approximation:
    - Lagrange and Newton interpolation polynomials, Chebyshev polynomials, Padè approximation, Splines
- Numerical Differentiation and Integration
    - Newton-Cotes quadrature rules, Gaussian quadrature

- Solution of Differential Equations
    - One-step methods for ODEs, Predictor-corrector schemes, Multi-step methods, Errors and stability,
    - Two-point boundary value problems, Shooting method, Finite difference method, convergence and stability

MA 537 updated November 2002.
MA 522, MA 625 updated August 2002.

Preliminary Examination in Partial Differential Equations

Partial Differential Equations (PDE) are studied using advanced calculus and real analysis. This examination tests the student's basic knowledge of the relevant tools from these subjects together with his/her ability to apply them to problems in PDE. The first half of the two-semester course in PDE covers the basic theory, including: the basic theory of first order quasilinear equations, classification of second order equations, the Cauchy problem, and the basic theory of the three “typical” elliptic, parabolic, and hyperbolic equations from mathematical physics (Laplace, heat, wave equations). The second half of the PDE sequence covers the “modern” theory of second order elliptic equations.

Basic Courses: MA 533, MA 633**.**

Suggested texts:

Partial differential equations, L.C. Evans, American Mathematical Society, 1998: Chapters 1-3, §4.6, Chapters 5, 6.

Elliptic Partial Differential Equations of Second Order, Second Edition, D. Gilbarg and N. Trudinger, Springer-Verlag 1983: Chapters 2-8.

Partial Differential Equations, Fourth Edition, John Fritz, Springer-Verlag 1981: Chapters 1, 3, 4, 6, 7.

Outline:

- **Basic Theory**
    - First order equations; method of characteristics
    - Power series
    - Noncharacteristic surfaces
    - Cauchy-Kowalevski Theorem
    - Laplace's equation
        - Fundamental solution
        - Mean-value properties
        - Properties of harmonic functions
        - Green’s function
        - Perron's solution of the Dirichlet problem
    - Heat equation
        - Fundamental solution
        - Mean-value properties
        - Properties of solutions
    - Wave equation

        - Method of spherical means
        - The non-homogeneous problem
- **Elliptic Equations of Second Order**
    - Sobolev spaces
        - L^p spaces
        - Weak derivatives
        - Approximation by smooth functions
        - $W^{k,p}$ spaces
        - Extension and traces
        - Sobolev and Morrey inequalities
        - Compactness
    - Existence of weak solutions
    - Regularity
    - Maximum principles
    - Harnack's inequality
    - Eigenvalues

Revised: 14 October 2002

Preliminary Examination in Topology

Basic courses: MA551, MA651.
Suggested text: In *Topology*, 2nd edition, by James R. Munkres, this list corresponds roughly to sections 12-35 (including 22), 37, 39, 41, 43-46, 48, 51-55, 58-60, 70
Outline:

- Topological spaces and continuous functions, including metric spaces, quotient spaces, product spaces, and uniform continuity.
- Connectedness and related concepts like local connectedness and path connectedness.
- Compactness and related topics like local compactness, paracompactness, and the Tychonoff theorem.
- First and second countability axioms.
- Separation axioms.
- Urysohn's lemma, the Tietze extension theorem, and the Urysohn metrization theorem.
- Complete metric spaces, including Cauchy sequences and compactness in metric spaces.
- The compact-open topology on function spaces.
- The Baire category theorem.
- The fundamental group, covering spaces, and the relations between them.
- The fundamental group of a circle and the Brouwer fixed-point theorem for the disc.
- The fundamental group of the torus and the projective plane.
- Seifert-van Kampen’s theorem.
- Homotopy type.

Updated August 2002.

Important Rules and Policies

The SACS 18 Hour Rule

The University of Kentucky is bound by rules of its accrediting association, the Southern Association of Colleges and Schools (SACS). One of these rules is that a graduate teaching assistant who has primary responsibility for teaching a class must have 18 graduate hours in mathematical sciences. (This rule does not apply to students who conduct recitation sections for large lecture classes.) Because of this rule, it is important that students complete 18 SACS acceptable credits before the start of their second year. SACS-acceptable courses include:

- Any MA/CS/STA course numbered 500 or above.
- Courses with the above prefixes and a G suffix (for example, MA471G).
- Any courses outside MA/CS/STA which are acceptable for the student's graduate degree (example PHY 416G).

Changing Courses

Supported graduate students may not change their course schedules (dropping or adding, changing between credit and audit) without the consent of the Director of Graduate Studies. In part, this is to insure that changes do not delay the student's degree, meeting the SACS 18 Hour Rule, or the completion of Preliminary Examinations.

Teaching Assistants submit their schedule of graduate classes to Mr. Tom Moss prior to the start of the semester; he uses this schedule to make their teaching assignments. Once this is done, it is virtually impossible for students to change their schedule, unless the change causes no conflict with their teaching.

Outside Employment

The Mathematics Department awards financial aid, (Teaching Assistantships, Fellowships, Research Assistantships) as part of a program to train students to become professional mathematicians. We expect that students will devote their full energies to this process. Hence, supported students should have the consent of the Director of Graduate Studies before accepting other employment, such as tutoring. This consent will require a demonstration of financial need and of the availability of the time to meet all the student's responsibilities.

Integrity

Graduate students are apprentice professionals. One of the most valuable possessions that professionals have is their integrity. Be careful not to compromise it. An undergraduate can sometimes break the rules and have it treated as- a minor offense; graduate students are held to a much higher standard.

One very important rule is that against plagiarism - using someone else's work or ideas as your own, without proper acknowledgment. For mathematics students, this issue most often arises in connection with homework assignments and take-home examinations. Teachers differ on how much outside help is permissible on a homework assignment. Some encourage student cooperation, others totally forbid it. *Be Sure that You Understand Your Teacher's Rules.* Is it all right to talk to other students in the course? to former students? to look at exercise sets from previous semesters? to consult outside books or faculty? On take-home examinations, outside help is almost never permissible.

The University Student Code specifies that the minimum penalty for plagiarism is a grade of **E** for the course; such a grade cannot be removed by using a repeat option.

Deadlines

There are some deadlines which are University or Graduate School rules, and must be followed. These may be found in the University Calendar which is published in the schedule of classes each semester or at www.uky.edu/Registrar/newhome/CALENDAR.html. In addition, the rules of the Graduate School are detailed in the Graduate School Bulletin. The Graduate Secretary will assist you in meeting deadlines and making the forms full throughout your career.

Getting Help

Academic Help

Help with course work is most easily available from other students, either current or former students in the class. Most people are comfortable about asking other students for help. Of course, one must be careful not to violate the teacher's rules on outside assistance; see earlier remarks on Integrity.

Students should feel free to ask their professors for help outside of class. Drop by their office or ask them a question in the Coffee Room. No one should feel that these questions are an imposition; such informal consultations are part of what professors are paid to do. Moreover, many faculty look forward to and enjoy these conversations.

Personal Problems

If students have personal problems which are affecting their studies or their teaching, they should tell someone. Good people to tell are their advisors, a teacher, or the Director of Graduate Studies. Students need not reveal any more details than they are comfortable telling. One advantage of notifying someone of the existence of a problem is that they can sometimes suggest a recourse. Also, it is easier to accept that personal problems have affected course performance if the student tells someone before an academic catastrophe occurs.

Financial Problems

The Department has a small fund which can sometimes loan TA's and RA's amounts up to $1,000. We expect to be repaid fairly rapidly from the student's stipend. There is no interest charge.

Financial Support

Renewal of Assistantships

Each year in January, the Graduate Committee evaluates all continuing graduate students. This evaluation serves as a basis for a recommendation to the faculty on whether the student should continue to receive financial support. The Committee uses all the information it can obtain about the student, including: grades in courses, evaluation of students given by teachers, performance on Preliminary Examinations, and teaching evaluations. Students are notified by March 1 of the faculty decision on their continued support.

Students in Master's programs (M.A. or M.S.) will ordinarily not be supported for more than two years.

Probation

Any graduate student who has accumulated 12 graduate credits and whose GPA is less than 3.0 is placed on probation by the Graduate School. Students on probation will not receive teaching assistantships. They also cannot receive tuition scholarships, and hence must pay their own tuition.

Summer Support

The Mathematics Department can usually offer summer support to most, but not all, of the students who request it. In particular, we are unable to support as summer TA's any student who has not satisfied the SACS 18 Hour Rule. Students in Master's programs will be given lower priority for Summer Teaching Assistantships. Factors which are considered in summer support decisions include:

- course grades;
- performance on Preliminary Examinations;
- student's degree program;
- student's dependents;
- performance as a teacher.

In early April, we notify the applicants for summer support that they have been placed in one of three categories:

- those who will definitely be supported;
- those on the waiting list;
- those who will not be supported.

Whether a person on the waiting list is supported depends primarily on the number of undergraduates enrolling in summer School. Frequently, this is unknown until shortly before the opening of classes.

Tuition Scholarships

Teaching Assistants usually receive tuition scholarships to pay both the in-state and out-of-state portion of graduate tuition. The tuition for research assistantships is to be paid by the granting agency. These tuition scholarships are ***not*** "waivers." Real state-appropriated dollars are used to pay them. If students drop a course early in the semester, so that they are no longer taking 9 credits, then the Chancellor's scholarship account can get part of the in-state money back. However, this process is not automatic; just because the Registrar's VIP system knows that a student has dropped doesn't mean that Student Billings knows. Each student should inform the Director of Graduate Studies of their add/drop and credit/audit transactions.

Fees

Currently enrolled students must pay a $50 Registration Confirmation Fee approximately 3 weeks before the beginning of the next semester. Students on tuition-scholarships must pay this fee; the amount will be applied toward the payment of the mandatory Student Health Fee and the campus recreation fee. There are no confirmation fees required for summer school. However, all currently enrolled students are expected to register before the first day of class for each summer session in order to avoid a $40 late registration fee. If you decide later that you wish to cancel your summer registration, you may do so using VIP without accruing any additional fees. You may cancel your summer registration at any time prior to the first day of the course(s) for which you have registered.

Full-time vs. Part-time Students

There is no single definition of "full-time" which covers all possible cases. Among the various rules are:

- A graduate student taking at least 9 credits pays (or has a tuition scholarship for) full tuition. There is no additional tuition for taking more than 9 credits.
- A Teaching Assistant taking at least 6 credits does not have FICA tax withheld (this is the social security tax).

- Students taking less than 9 credits are not eligible for an Activity Card. This means they cannot purchase basketball or football tickets, or use sports facilities.
- The requirements for “full-time student” for someone who is deferring repayment of a federally-managed student loan vary with the program. Check with the supervising agency.

Preparing for a Professional Career

A career as a professional mathematician usually requires taking courses, fulfilling requirements, and having a particular advanced degree. But these necessary conditions are rarely sufficient for professional success. While in graduate school, students should also do other things which will help them professionally. Many of these activities are naturally and freely available; getting the most from them usually calls for some initiative on the part of the student. Here are some things you can do:

Become an Active Part of the Community. Undoubtedly, the greatest resource the Department has to offer is simply all of the people who compose it. Through informal conversations with students and faculty, each of us can learn not only about mathematical content or how to teach a certain topic, but also about the issues that the profession faces, or what is going on elsewhere. Talk to people, ask questions, tell your experience, offer your opinion.

Attend Seminars and Colloquia. Listening to these will not only increase your knowledge of mathematics, but will also let you learn what mathematical questions interest a particular faculty member. This is one way that you learn who you might want to work with. Students will also find interesting seminars in other departments and the Center for Computational Sciences.

Once beyond your first year, you should also volunteer to talk yourself. We all have to stand in front of groups of peers and explain what we've been doing. Talking to the Graduate Student Colloquium is a good place to start this process.

If there is an advanced talk by a visitor, and you're afraid that you might not understand it, go anyway. Take along some paper and a problem to think about. If you're totally lost after 15 minutes, you can sit and think about your problem while the speaker finishes.

Read the Professional Journals. The American Mathematical Society, The Mathematical Association of America, The Operations Research Society of America, The Society for Industrial and Applied Mathematics all have extensive publications. Many of them are accessible and of interest to graduate students. They can be found in the coffee room and the mathematics library. Among the journals of general interest are *The Notices* and *The Bulletin of the American Mathematical Society, The American Mathematical Monthly* and *Focus* published by the Mathematical Association of America (MAA) and *SIAM News* and *SIAM Review* published by the Society for Industrial and Apples Mathematics.

Attend Professional Meetings. The Annual Meeting of AMS/MAA is held in early January. In addition, each year there are two meetings held in Kentucky:

The Kentucky Section of the MAA meets each year in April, and the Kentucky Mathematical Association of Two-year Colleges meets in early March, at Shakertown. The MAA meeting is primarily concerned with teaching mathematics in college; the KYMATYC meeting is a good one to go to if you're looking for a teaching job in a community college. Regional meetings such as the Midwest Partial Differential Equations Seminar and other subject related meetings provide a good opportunity to meet nearby experts in your research area. The Graduate School will provide some financial support for graduate students who wish to travel to professional meetings to present a paper. Students may attend conferences supported by the IMA using funds made available through the department's membership in the IMA.

The Academic Job Market

The academic job market for university mathematicians varies. The strength of the job market depends on many factors. A weak economy usually leads to budget cuts by state governments and in turn to a reduction in hiring of academic mathematicians. Current graduate students will be looking for jobs between one and five years from now and it is difficult to predict the demand for mathematicians five years in advance.

Students should keep themselves informed about the job market. Two good sources are: *The Annual AMS-MAA Survey*, which appears in the *Notices of AMS*, usually in November with a follow-up in July/August.

Students who plan to teach in universities, colleges, community colleges or high schools should begin in graduate school to build a strong resume. What follows is a list of some things to do.

- Teach a wide range of courses as a TA. Prospective employers want teachers who haven't spent five years teaching just algebra and trigonometry.
- Take the opportunity to teach any special courses, such as Math Excel, and the Freshman Summer Program. Assist in course development if asked.
- Be able to document your teaching performance. All teachers of mathematics courses are evaluated by their students every fall and spring. Unfortunately, the University-required form for this evaluation yields only average numbers provided by students checking boxes. If we want to improve our teaching, and to demonstrate teaching ability to outsiders, a different kind of evaluation form is needed. It should provide for narrative responses by students. TA's are urged to write their own forms, use them regularly, and retain them for eventual use by prospective employers.
- Acquire a faculty teaching mentor. This should be someone who will watch you teach and offer suggestions on how you can improve. Such a person can also write a letter of recommendation based on personal knowledge of your teaching.

Effective Studying

Study Tips for the Mathematics Graduate Student *Joy Denise Williams*

As a first-year graduate student, you have already proven yourself successful at doing undergraduate mathematics. The critical thinking skills you have developed will benefit you greatly in your graduate courses. However, you may find that in order to do well at the graduate level, you must develop new methods of studying that differ from those you used as an undergrad. Following are some of the studying techniques that have served me well as a graduate student

I

Studies have shown that students are more apt to retain information from lectures if they review the material immediately after class. I have found the best way to review is to recopy my lecture notes. This tactic has without doubt been the single most beneficial studying method that I have used here at graduate school. By writing the material down in my own words, while it is still fresh in my mind, I get a much firmer grasp on the concepts. While I am recopying, I will often come across some idea that I just do not understand. The worst thing to do at that point is to decide the concept really wasn't that important anyway and simply forget about it. First of all, if the professor mentioned it during his/her lecture, it is pretty safe to assume that it is important. Secondly, when it comes time to start preparing for an exam, all of those questions that went unanswered during the semester will resurface. My suggestion is to write down your questions while you are recopying your notes and then see the professor about them as soon as possible. Please, do not feel timid about going to see a professor. One thing I have found here at UK is that the professors encourage their students to stop by if they have questions.

Something else you might consider doing is setting aside time each day to re-read a portion of your lecture notes. What I generally do before every class is to read over my notes from the previous two or three lectures. Then when I go into class, my mind is already focused on the current topic of study, so I am more apt to follow what the professor is saying from the very beginning of his lecture.

II

In many, if not all of your pure math courses, a great emphasis will be placed on proofs. Professors will present proofs throughout the semester and will expect students to work through those proofs, or ones similar to them, on an exam. For this reason, I keep a separate notebook that contains just proofs. I update this notebook after every lecture; this is done concurrently with recopying my class notes. I then periodically review the proofs so that when the time comes to prepare for the exam, I am already familiar with them. You will be surprised at how many proofs a professor will do in the course of a semester. If you wait until a week or so before the exam to begin studying them, you may find yourself overwhelmed with the amount of material.

At this point, you might be wondering what it means to "periodically review proofs." Different people have different methods of studying proofs, but following is a technique that has worked well with me. Every morning, I select three or four proofs from my notebook and attempt to work through them on my own. Of course, if I get stuck, I have my notebooks there as a reference. I usually set aside between 30 minutes and an hour each day to do this. By actually working through the proofs rather than simply reading over them, I gain a deeper insight into the underlying concepts.

III

A skill I have developed here at graduate school and one that has proved to be very beneficial is working together with other students. As an undergraduate, I always worked by myself, and I realize now that I missed out on a lot because of it. This year, about eight of us first year students formed a study group for one of our classes. Every Friday afternoon, we met in a conference room for approximately two hours and discussed the notes we had taken in class during the week. I found this kind of environment to be very conducive to learning, for everyone had a chance to ask questions as well as to answer them. The greatest test of understanding of a concept is being able to explain it to other people, and this setting gave each person in the group the opportunity to do just that.

Also, feel free to work together in groups on homework assignments, unless of course the professor specifically asks you not to. Again, this is something I never did before coming to graduate school but wish I had started doing earlier. I think one of the most difficult things about doing math assignments is getting started, and working in groups has made this easier for me. Something I found especially helpful is getting together with other students soon after receiving an assignment to "throw around ideas." That is, we look at each problem and discuss possible strategies for solving it. Some people will actually work the problems together; what I generally do is to go someplace quiet immediately after the initial group meeting, and work on the problems by myself. With all those ideas still fresh in my mind, I usually have no problem getting started on a solution.

IV

The last topic I would like to address is how to prepare for an exam. Many people dread studying for exams and thus they will put it off as long as possible. This technique may have worked in some of your undergraduate courses, but it will not work at graduate school. The reasons for this are as follows: in each of your graduate courses, you will generally have only two exams - one mid-term and one (cumulative) final. You exams scores will constitute a very large if not entire percentage of your grade for the course, so it is important that you do well on both of them. (This is much different from those undergraduate courses where you had an exam every couple of weeks, and one bad test score really didn't harm your overall grade.) Also, “cramming” is completely ineffective because of the quantity of material is so great. By having only two exams, you will be held responsible for a large amount of information on each one.

If all of this is beginning to scare you, don't let it! For the good thing is that if you follow my first three suggestions for studying, I think you will find that preparing for an exam is the easy part! By recopying and re-reading your lecture notes, periodically working through proofs, and reviewing notes with other students, you will essentially be studying for the exam continuously, throughout the entire semester. Therefore, by the time the exam approaches, you will already have a firm grasp on the material. For me, the time right before exams is the least stressful time of the semester, because I don't feel overwhelmed with the material over which I will be tested. All I generally do to formally “study for an exam” is to read through my lecture notes and textbook, re-work problems done in class and in previous homework assignments, and work through the proofs in my “proof notebook.” I think you will be amazed at how much of your notes you will remember simply because you took the time to recopy them!

The techniques outlined in these pages have served me well as a graduate student. I hope they will be of help to you as you pursue an advanced degree in mathematics.

Doing Graduate Mathematics *Frank Branner*

I did not get the following advice in August, 1983, which proves that I will not have had access to an affordable time machine during the rest of my life. Had this been possible, I might have saved countless hours by managing my studies using the notions listed below.

Be single-minded in your effort to pass the prelims. Two things are necessary: 1) Get a complete understanding of the core subjects; and 2) Become expert at doing exercises and solving problems. Single-mindedness means, among other things, that you should shun all suggestions to take courses not directly related to the core subjects, and you should unrelentingly look for ways to reinforce your basic knowledge. This could mean taking review courses in areas that you feel weak in, *e.g.* linear algebra, advanced calculus, etc. (Do not let pride keep you from looking back at things you studied before. No pro golfer would think of not spending many hours each week on the practice green and driving range to keep sharp.)

- Understand the Core Subjects.
- Preread Lecture Topics.

Get a schedule of lecture topics, from the syllabus if it is detailed enough, or from the teacher at the close of each session. Read the material before the classroom presentations, memorize the definitions and the theorem statements, and identify the hard parts in the proof. The failure to follow lectures often results from an incomplete understanding of the fine points of definitions and theorems, and this kind of understanding is not easy to get during the first hearing, especially if you feel the pressure of taking complete notes. Also, knowing what is difficult for you ahead of time will give you the chance to ask the teacher for help, something you couldn't do if you lost track of the proof at the start. Losing track of a proof during a lecture is a very bad but easy thing to do and a very bad habit to fall into. It leads to boredom and to broken promises to oneself to catch up later.

Previewing the topic should help you to avoid the attention deflecting activity of taking voluminous notes. You can write down only those ideas not in the textbook, and listen better. If you are able to sell the idea of prereading to the other students, you may be able to move the teacher away from the format of a mere recitation of definitions, theorems, and proofs already available in the book to more useful things like proof strategy, utility, and discussion of exercises and problems.

Review First Things Repeatedly.

Review repeatedly, at least once a week, the first chapter of the textbook, the first section of each chapter studied, and the first paragraphs of each section. This is where the basic definitions, examples, and theorems reside, and you must know them better than the alphabet, the multiplication table, even better than your name.

***Study The Examples*.**

The examples that are given to illustrate definitions are the bridges connecting sometimes obscure abstractions and the more concrete real world. Understanding the examples will reinforce your grip on the definitions and will provide a good framework for thinking about problems.

Play With The Theorems.

Identify the structure and strategy of the proof. What general proof technique was used? What earlier results and definitions were used? (Mark in your book and notes at the places of this earlier work that it was used in the current proof.) How and where are the hypotheses used? Try to find counterexamples to show that the theorem fails if any of the hypotheses are omitted. Ask what potential uses this theorem might have, and try to make up problems related to it.

Don't Let Them Confuse You.

Do not allow any significant part of the subject to remain vague after the class lecture. It is easy to promise yourself that you will come back to that topic later, but your probably won't. Instead, give it another try yourself, and if this does not help, hound your fellow students and teacher for clarification. (By asking your fellows, you really do them a favor by getting them to explain and thus fix the idea better in their minds. Of course you must make yourself available for similar consultations.)

Don't Forget.

It is very easy, once you've complete a course, to stop thinking about it and give priority to present studies. Try to avoid this by looking for connections to your new work, or by setting aside definite review times.

Become An Expert Problem Solver.

Learn The Techniques of Mathematical Proofs.

Of course you already know about truth tables, negation, contrapositives, etc. But it will be very helpful to study these ideas a lot more. A quick recognition of the framework of a written proof should help you to read it faster, and a knowledge of generic approaches to proofs may help you to find the right way sooner. An interesting (small) book on this subject is *How To Read and Do Mathematical Proofs, An Introduction to Mathematical Thought Processes* by Daniel Solow, John Wiley and Sons. Why not organize an informal seminar to study this book and discuss its ideas relative to one of your current courses? While on the subject of books, Paul Halmos's book, *I Want To Be A Mathematician* gives a super view of graduate school life and times.

Do a Lot of Problems.

Work as many exercises in the book as you have time for. Read over the ones that you don't actually solve to expand your idea about the scope of the topic. Talk about problems and techniques to your fellow class members.

Study Old Prelims.

Get copies of all available prelim exams (available from Raul Camarillo, 767 Patterson Office Tower), and solve all of the problems on them. Do not memorize the solutions, but rather get an understanding of the ideas behind them and the clues that pointed to the successful solution method.

Campaign for More Focus on Problems.

Ideally all assigned homework would be discussed thoroughly during the next class period, but this hardly ever happens. Sometimes homework is returned only after weeks, sometimes never. Find some way to get around this difficulty. Try selling the department on using some (advanced) graduate students as “grading assistants”, or talk the teacher into assigning grading chores to class members on a rotating basis.

Graduate Courses in Mathematics

The graduate course offerings of any large department are in a constant state of flux: new courses are added, outdated ones are dropped, descriptions, titles and numbers are changed. Some courses are offered every year, others only infrequently. Thus, a list of courses without special information or a reader's guide can leave erroneous impressions. With that caveat, what follows is a current list of the Department's course offerings as a guide to understanding the preceding program descriptions. (All courses are 3 credit hours each unless otherwise specified.) The planned frequency of offering is given following the course name. If no frequency is specified, the course is offered irregularly, usually in response to student interest.

Note: Courses marked (MAT) are restricted to students enrolled in programs of the College of Education.

MA 501-502, *Seminar in Selected Topics* (MAT) - (3 hrs.each). Various topics from the basic graduate courses. Designed as a course for teachers of lower division mathematics and usually offered in connection with a summer institute. May be repeated to a maximum of 6 credits. Prereq: Teaching experience in the field of mathematics, or consent of instructor.

MA 506, *Methods of Theoretical Physics I*, every fall. This course and its sequel (MA/PHY 507) are designed to develop, for first-year graduate students, familiarity with the mathematical tools useful in physics. Topics include curvilinear coordinates; infinite series; integrating and solving differential equations of physics; and methods of complex variables. Work with Green's functions; eigenvalues; matrices and the calculus of variations are included as a part of MA/PHY 506 and 507. (Same as PHY 506.)
Prerequisite: PHY 404G or equivalent.

MA 507, *Methods of Theoretical Physics II*, every spring. Continuation of MA/PHY 506. Fourier and Laplace transforms; the special functions (Bessel, Elliptic, Gamma, etc.) are described. Work with Green's functions, eigenvalues, matrices and the calculus of variations are included as a part of MA/PHY 506 and 507. (Same as PHY 507.) Prerequisite: MA/PHY 506.

MA 515, *Mathematical Programming and Extensions*, every fall. Mathematical and computational aspects of linear programming; large scale structures; quadratic programming; complementary pivoting; introduction to nonlinear programming. Applications to engineering and economics. Additional topics selected in geometric programming, stochastic programming. (Same as STA 515.) Prerequisite: A course in linear algebra or consent of instructor.

MA 522, *Matrix Theory and Numerical Linear Algebra I,* every fall. Review of basic linear algebra from a constructive and geometric point of view. Factorizations of Gauss, Cholesky and Gram-Schmidt; determinants; linear least squares problems; rounding error analysis; stable methods for updating matrix factorizations and for linear programming; introduction to Hermitian eigenvalue problems and the singular value decomposition via the QR algorithm and the Lanczos process; method of conjugate gradients. (Same as CS 522.) Prerequisite: MA 322.

MA 527, *Applied Mathematics in the Natural Sciences I.* Construction, analysis and interpretation of mathematical models applied to problems in the natural sciences. Physical problems whose solutions involve special topics in applied mathematics are formulated, various solution techniques are introduced, and the mathematical results are interpreted. Fourier analysis, dimensional analysis and scaling rules, regular and singular perturbation theory, random processes and diffusion are samples of selected topics studied in the applications. Intended for students in applied mathematics, science and engineering. Prerequisite: MA 432G or three hours in an equivalent junior/senior level mathematics course or consent of the instructor. (Same as EM 527.)

MA 533, *Partial Differential Equations*, every fall. Elementary existence theorems, equations of first order, classification of linear second order equations, the Cauchy and Dirichlet problems, potential theory, the heat and wave equations, Green's and Riemann functions, separation of variables, systems of equations. Prerequisite: MA 532 and MA 472G or equivalent.

MA 537, *Numerical Analysis I*, every semester. Computer arithmetic. Discussion of the various types of errors, solution of systems of linear algebraic equations - Gaussian elimination with partial pivoting and sealing. Iterative refinement. Polynomial and piecewise polynomial interpolation. Orthogonal polynomials. Method of least-squares. Numerical differentiation. Numerical integration: Newton Cotes formulas and Gaussian quadrature. Prerequisite: MA/CS 321, or graduate standing, or consent of instructor. Knowledge of a procedural computer language is required. (Same as CS 537.) (In the spring, a section will be offered will be offered which is designed to prepare mathematics students for the preliminary examination in numerical analysis.)

MA 551, *Topology I,* every fall. Topological spaces, products, quotients, subspaces, connectedness, compactness, local compactness, separation axioms, convergence. Prereq: Consent of instructor.

MA 561, *Modern Algebra I,* every fall. Algebraic structures, quotient structures, substructures, product structures, groups, permutation groups, groups with operators, and the Jordan-Holder theorem. Prerequisite: Consent of instructor.

MA 565, *Linear Algebra*, every fall. Review of finite dimensional linear algebra, the rank of a matrix, systems of linear equations, determinants, characteristic and minimal polynomials of a matrix, canonical forms for matrices, the simplicity of the ring of linear mappings of a finite dimensional vector space, the decomposition of a vector space relative to a group of linear mappings and selected topics of a more advanced nature. Prerequisite: MA 322 or consent of instructor.

MA 570, *Multivariate Calculus.* A self-contained course in *n*-dimensional analysis, including the general form of Stokes' theorem. Prerequisite: MA 432 or equivalent.

MA 575, *Introduction to Analysis*. A first course in the rigorous analysis of functions of real variables. The real and complex number systems are developed and basic notions of topology and metric spaces are discussed. Sequences and series, continuous functions, differentiation of functions of a single real variable, and Riemann-Stieltjes integration are rigorously developed. Convergence, differentiation, and integration of functional series are discussed together with equicontinuity and the Stone-Weierstrass theorem.

MA 611, *Independent Work in Mathematics*, every semester. Reading course for graduate students in mathematics. Prerequisite: Major in mathematics, a standing of at least 3.0 and consent of instructor. May be repeated to a maximum of nine credits.

MA 613, *Problems Seminar in Operations Research*. In this course the student is exposed to the art of applying tools of operations research to real world problems. The seminar is generally conducted by a group of faculty members from the various disciplines to which operations research is applicable. Prerequisite: MA 617 and STA 525 or consent of the instructor. (Same as BA/EE/STA 619.)

MA 614, *Algebraic Combinatorics*, every spring. An introduction to the basic notions and techniques in enumerative combinatorics. The material has applications to polytopal theory, hyperplane arrangements, computational commutative algebra, representation theory and symmetric functions. Topics include generating functions, the principle of inclusion and exclusion, bijections, recurrence relations, partially ordered sets, the Mobius function and Mobius algebra, the Lagrange inversion formula, the exponential formula and tree enumeration. Prereq: A graduate course in linear algebra or consent of instructor.

MA 618, *Combinatorics and Networks*, every spring. Graphs; networks; min flow-max cut theorem and applications; transportation problems; shortest route algorithms; critical path analysis; multi-commodity networks; covering and packing problems; integer programming; branch-and-bounding techniques;

cutting plane algorithms; computational complexity. Prerequisite: MA 515, can be taken concurrently with MA 515.

MA 622, *Matrix Theory and Numerical Linear Algebra II*, every spring. Numerical solution of matrix eigenvalue problems and applications of eigenvalues; normal forms of Jordan and Schur; vector and matrix norms; perturbation theory and bounds for eigenvalues; stable matrices and Lyapunov theorems; nonnegative matrices; iterative methods for solving large sparse linear systems. (Same as CS 622.)

MA 625, *Numerical Methods for Differential Equations* Numerical solution techniques for boundary value problems for ordinary differential equations, and for parabolic and elliptic partial differential equations. Prerequisite: CS/MA/EGR 537 or consent of instructor.

MA 628, *Applied Mathematics in the Natural Sciences II.* Continuation of MA/EM 527 with emphasis on special topics and techniques applied to partial differential equations that occur in various physical field theories. Field equations of continuum mechanics of solids and fluids are reviewed. The method of characteristics; elliptic functions and integrals, Legendre polynomials; Mathieu functions; integral equations and transforms; and the methods of potential theory are examples of selected topics studied in introductory applications. Intended for students in applied mathematics; science and engineering. Prerequisite: MA 527

MA 630, *Mathematical Foundations of Stochastic Processes and Control Theory I.* A modern treatment of Stochastic Processes from the measure theoretic point of view; the basic notions of probability theory; independence; conditional expectations; separable stochastic processes; martingales; Markov processes; second-order stochastic processes.
Prerequisite: MA 432 or MA 571.

MA 633, *Theory of Partial Differential Equations*, every spring. A continuation of MA 533. Topics may include hypoelliptic operators and interior regularity of solution; P(D) - convexity and existence theorems, regularity up to the boundary; applications of the maximum principle; semi-group theory for evolution equations; perturbation methods, well- posed and improperly posed problems; equations with analytic coefficients; asymptotic behavior of solutions; nonlinear problems. Prerequisite: MA 533.

MA 641, 642, *Differential Geometry.* Tensor products; exterior algebra; differentiable maps; manifolds; geodesics; metric properties of curves in Euclidean space; fundamental forms; surfaces.
Prerequisite: Consent of instructor.

MA 651, *Topology II*, every spring. Embedding and metrization; compact spaces; uniform spaces and function spaces. Prerequisite: MA 551

MA 654, *Algebraic Topology I*, fall odd numbered years. Homotopy and homology theories; complexes and applications. Prerequisite: MA 561 and MA 651, or equivalent.

MA 655, *Algebraic Topology II*, spring even-numbered years. Singular homology theory and applications; homology of products; singular and Cech cohomology with applications. Prerequisite: MA 654.

MA 661, *Modern Algebra II*, every spring. Rings; fields of quotients; rings of polynomials; formal power series; modules; exact sequences; groups of homomorphisms; natural isomorphisms; algebras and tensor algebras. Prerequisite: MA 561 or consent of instructor.

MA 667, *Group Theory*. A study of homomorphisms for groups; finite groups; solvable groups; nilpotent groups; free groups; and abelian groups. Prerequisite: MA 661.

MA 671, *Functions of a Complex Variable I*, every spring. Differentiation and integration, contour integration, poles and residues. Taylor and Laurent series, and conformal mapping. Prereq: MA 575 or consent of instructor.

MA 672, *Functions of a Complex Variable II*, every fall. A continuation of MA 671 to include the Riemann Mapping theorem; Dirichlet problem; multiple valued functions; Riemann surfaces and applications. Prerequisite: MA 671.

MA 676, *Analysis I*, every spring. Sequences and series of real and complex numbers, sequences of functions. Riemann-Stieltjes integration, Lebesgue measure and integration. Prerequisite: MA 575 or consent of instructor.

MA 677, *Analysis II*, every fall. Continuation of MA 676. Absolutely continuous functions on the real line; L^p-spaces; beginning theory of Banach spaces including the Hahn- Banach; closed graph; and open mapping theorems. Prerequisite: MA 676 or consent of instructor.

MA 681, *Functional Analysis I*. General theory of normed linear spaces including the Hahn-Banach separation theorems; principle of uniform boundedness and closed graph theorem; dual spaces and representation theorems for linear functionals; abstract measure theory and Riesz representation theorem for $C(X)$.Prerequisite: MA 676 or consent of instructor.

MA 682, *Functional Analysis II*. Weak and weak-star topologies; reflexive spaces; convexity in linear spaces; operators; adjoint operators; spectral theory and Fourier transforms.
Prerequisite: MA 681.

MA 714, *Selected topics in discrete mathematics*. Review of recent research in discrete mathematics. May be repeated to a maximum of nine credits. Prerequisite: Consent of instructor.

MA 715, *Selected Topics in Optimization*. Topics will be selected from the areas of mathematical control theory; integer programming; combinatorial optimization; large-scale optimization; nonlinear programming; dynamic optimization, etc. May be repeated for a maximum of nine credits. Prerequisite: Consent of instructor.

MA 721, *Selected Topics in Numerical Analysis*. Review of current research in numerical analysis. May be repeated to a maximum of nine credits. Prerequisite: Consent of instructor.

MA 732, *Selected Topics in Differential and Integral Equations.* Advanced topics in theory of differential (ordinary or partial) and integral equations such as topological dynamics; almost periodic solutions; stochastic differential equations; integro-differential and differential-difference equations; generalized functions as solutions; non-linear partial differential equations; singular equations.

MA 748, *Master's thesis research.* Half-time to full-time work on thesis. May be repeated to a maximum of six semesters. Prereq: All course work toward the degree must be completed.

MA 749, *Dissertation Research*. Half-time to full-time work on dissertation. May be repeated to a maximum of six semesters. Prerequisite: Registration for two full-time semesters of MA 769 residence credit following the successful completion of the qualifying exams.

MA 751, 752, *Selected Topics in Topology* - (3 hrs. each). Prerequisite: MA 651.

MA 761, *Homological Algebra* Homological algebra; modules; exact sequences; functors; homological dimension; extension problems. Prerequisite: Consent of instructor.

MA 764, 765, *Selected Topics in Algebra* - (3 hrs. each). Reports and discussion on recent advances in groups theory; ring theory; and homological algebra. Prerequisite: MA 661 and consent of instructor.

MA 768, *Residence Credit for Master's Degree.* (1-6 hours), May be repeated to a maximum of 12 hours.

MA 769, *Residence Credit for Doctoral Degree* May be repeated indefinitely. (1-12 credits equivalence) May be repeated indefinitely. (Students should register for two semesters of nine hours residency credit or three semesters at 6 hours. With the approval of the students advisory committee, the Director of Graduate Studies and the Graduate Dean, students may substitute course work for part of the residency requirement.)

MA 772, Selected Topics in the Theory of Complex Variables Prerequisite: Consent of instructor.

MA 773, 774, *Selected Topics in Analysis* May be repeated once for a maximum of six credits. Prerequisite: Consent of instructor.

MA 777, 778, *Mathematical Seminar* May be repeated once for a total of six credits. Prerequisite: Consent of instructor.

Courses in Other Departments

Some courses offered by other departments, which are of interest to Mathematics graduate students, are listed below.

CS 505, *Database Management Systems*, every semester This course is primarily concerned with the definition, organization, and manipulation of a database. An overview of the goals of database management is examined. The database management process is broken down into its four constituent parts: data definition; data manipulation; data retrieval; and report generation. Attempts toward standardization in database management are presented with emphasis on the CODASYL activities. The concept of shared files and the deadlock systems are surveyed, and at least one case study is examined in detail. Prerequisite: CS 370. Restricted to Computer Science and Electrical Engineering majors. Others by permission.

CS 673, *Error correcting codes,* The problem of correct transmission of data in a noisy environment. The design and analysis of codes that efficiently (in terms of data rate and encryption and decryption speed) correct errors. Linear and nonlinear block codes, general encoding and decoding techniques, fundamental bounds, dual codes, cyclic codes. Specific codes will be studied, including Hamming, BCH, Reed-Muller, Reed-Solomon, trellis, and convolutional codes. Prereq: CS 515 or consent of the instructor.

CS 678 *Cryptography*. The study of security in communications and electronic computing. The encryption of data using public key systems, block ciphers, and stream ciphers. The basic tools for the design and analysis of such systems. Topics may include information theory, authentication, digital signatures, secret sharing schemes, complexity theoretic issues, probabilistic encryption, electronic commerce and others. Prereq: CS 515 or consent of the instructor.

ECO 703, *Introduction to econometrics, I,* (3 hours). The first course in the introduction to econometrics. A comprehensive survey of the general linear regression, autocorrelation, errors in variables and distributed lag models. Prereq: ECO 603, STA 424G, STA 525 or consent of instructor.

ECO 706, *Introduction to econometrics, II. (3 hours).* The second course in the introduction to econometrics. A comprehensive survey of identification, estimation and hypothesis testing in the context of simultaneous equations model. Prerequisite: ECO 703 or consent of instructor.

CS/EE 635, *Image processing,* every semester (3 hours). The course outlines applications of image processing and addresses basic operations involved.

Topics covered include image perception, transforms, compression, enhancement, restoration, segmentation, and matching. Prereq: Graduate standing and consent of instructor.(Same as CS 635.)

FR 011, *French for Reading Knowledge*, every fall and summer This course is designed to meet the needs of upper-division and graduate students who are preparing for the graduate reading examination.

GER 011, *German for Reading Knowledge*, every semester This course is designed to meet the needs of upper-division and graduate students who are preparing for the graduate reading examination, who need a reading knowledge of German in their minor, or who require a review of German grammar.

STA 524, *Probability*, every fall, sample space; random variables; distribution functions; conditional probability and independence; expectation; combinatorial analysis; generating functions; convergence of random variables; characteristic functions; laws of large numbers; central limit theorem and its applications. Prerequisite: MA 532G or 571G or consent of instructor.

STA 525, *Introductory Statistical Inference*, every spring Simple random sampling; statistics and their sampling distributions; sampling distributions for normal populations; concepts of loss and risk functions; Bayes and minimax inference procedures; point and interval estimation; hypothesis testing; introduction to nonparametric tests; regression and correlation. Prerequisite: MA/STA 320 or STA 524 or STA 424G.

STA 624, *Applied Stochastic Processes,* every spring, Definition and classification of stochastic processes; renewal theory and applications; Markov chains; continuous time Markov chains; queuing theory; epidemic processes; Gaussian processes. Prerequisite: STA 524 or consent of instructor.

STA 626, *Time series analysis,* (3 hours). Time series and stochastic processes, auto-correlation functions and spectral properties of stationary processes; linear models for stationary processes, moving average, auto-regressive and mixed autoregressive-moving average processes; linear nonstationary models, minimum mean square error forecasts and their properties; model identification, estimation and diagnostic checking. Prereq: STA 422G or equivalent. (Same as ECO 790.)

Departmental Expectations for Graduate Students in Mathematics at the University of Kentucky

First Semester	At a minimum, complete two graduate mathematics courses with a B average (3.0).
First Year	At a minimum, complete five graduate courses in mathematics (or mathematically related subjects). By now, the grade point average should be above a B average (3.16, say). Doctoral students should attempt a preliminary exam.
Second Year	At the end of their second graduate year, all students should hold a master's degree (either earned at U.K. or previously awarded). Continuing Ph.D. students may choose to delay final exam until early in the fall semester of their third year. Students should now have a grade point average of at least 3.25. Doctoral students should now have passed at least one preliminary exam. Doctoral students should now have a 3.5 grade point average.
Third Year	Doctoral students should now have passed all three preliminary exams.
Fourth Year	Doctoral students should now have formed their doctoral committees and passed their qualifying exams.
Fifth, Sixth Years	Master a field, prove some theorems, write and defend a dissertation. Find a job.

Students who enter U.K. with previous graduate experience should exceed these expectations. Here is a timeline for the doctoral student who wants to speed up the process.

An Ideal Timeline for a Doctoral Student

First Year	Complete six graduate mathematics courses with a grade point average of at least 3.3. Pass one preliminary exam.
Second Year	Have passed all three preliminary exams.
Third Year	Have formed the doctoral committee and passed the qualifying exam.