The Second Ohio River Analysis Meeting

The University of Kentucky, Lexington, Kentucky

April 21-22, 2012

TITLE AND ABSTRACT

Main Speakers:

(**1**) Professor Patti Bauman, Purdue University

Title: *Analysis of Equilibria with One-Half Degree Defects for the Landau-de Gennes Model of Nematic Liquid Crystals*

Abstract: We investigate the structure of nematic liquid crystal thin films described by the Landau–de Gennes tensor-valued order parameter model with Dirichlet boundary conditions on the sides of nonzero degree. We prove that as the elasticity constant goes to zero in the energy. a limiting liquid crystal uniaxial texture forms with a finite number of defects, all of degree one-half or all of degree minus one-half, corresponding to vertical disclination lines at those locations. We also describe a result on the limiting behavior of minimizers of the Chern-Simons-Higgs model without magnetic field that follows from a similar proof.

(**2**) Professor Alice Chang, Princeton University

Title: *Higher order isoperimetric inequalities –an approach via method of optimal transport*

Abstract: One of the method to derive sharp isoperimetric inequality for domains in the Euclidean is to apply the method of optimal transport; in this talk, I will report some recent joint work with Yi Wang to extend the method to prove some higher order isoperimetric inequalities with weights involving symmetric functions of the second fundamental form.

(**3**) Professor Michael Frazier, University of Tennessee, Knoxville

Title: *Global Estimates for Kernels of Neumann Series and Green's Functions of Schrödinger Operators*

Abstract: This work is joint with Fedor Nazarov and Igor Verbitsky. Consider a linear operator T with operator norm $\|T\| < 1$, and the Neumann series $I + T + T^2 + \dots$ for $(I - T)^{-1}$. Suppose T is an integral operator on $L^2(\Omega)$, where Ω is a σ-finite measure space. Suppose the kernel K of T is non-negative, measurable, symmetric, and satisfies a certain quasi-metric condition (example: the Riesz potential). We show that the kernel H of $S = T + T^2 + T^3 + \dots$ is bounded below by $Ke^{c_1 K_2/K}$ and above by $Ke^{c_2 K_2/K}$, where K_2 is the kernel of T^2, for constants $c_1, c_2 > 0$. We apply this result to obtain estimates for Green's functions associated with (possibly fractional) Schrödinger operators $(-\triangle)^{\alpha/2} - q$ for $q \geq 0$ and $0 < \alpha \leq 2$, on a domain Ω which could be all of $\mathbb{R}^n$, or a bounded domain in $\mathbb{R}^n$ which satisfies the boundary Harnack principle. These results can be restated as estimates for the conditional gauge of an α-stable process. As a consequence, we

obtain existence criteria and estimates for the Feynman-Kac gauge u_1, the solution of $-\triangle u_1 = qu_1$ on Ω and $u_1 = 1$ on $\partial\Omega$. Solvability criteria for a certain non-linear equation of Ricatti type can be obtained as an application.

(**4**) Professor Fanghua Lin, Courant Institute of Mathematical Sciences, New York University

Title: *Elliptic Equations with Periodic Coefficients and Theory of Homogenization*
Abstract: I shall first review some old results, then describe two problems that motivate some of the studies and, some recent results obtained in joint with C.Kenig and Z.W.Shen.

(**5**) Professor Balint Virag, University of Toronto, Canada

Title: *Limits of random Schroedinger operators and random matrices*
Abstract: In 1957 Wigner proposed to model energy levels atomic nuclei by eigenvalues of random matrices. Since then, several complex systems have been predicted by physicists to have random-matrix like behavior. Indeed, Dyson referred to random matrix theory as the "new statistical mechanics". I will review some models and show a very special case where this behavior can be mathematically proven.

(**6**) Professor Yu Yuan, University of Washington, Seattle

Title: *Lagrangian mean curvature flow for entire Lipschitz graphs*
Abstract: We talk about longtime existence and estimates for solutions to the fully nonlinear Lagrangian parabolic equation with initial potential satisfying: either a) the eigenvalues of the Hessian of the potential is almost less than one; b) the potential is convex; or c) the potential satisfies a large supercritical Lagrangian phase condition. This is joint work with Albert Chau and Jingyi Chen.

Contributed Talk Speakers:

(**1**). Murat Akman, University of Kentucky

Title: *On the logarithm of the minimizing integrand for certain variational problems in two dimensions*
Abstract: In this talk, I will discuss joint work with John Lewis and Andrew Vogel on minimizer of the certain variational problem for the functional whose integrand is $f(\nabla u)$ with $\nabla u \neq 0$ at z. We obtain that in a neighborhood of z, $\log f(\nabla u)$ is a sub solution, solution, or super solution for a certain pde respectively when $p > 2, p = 2, p < 2$.

(**2**). Professor Robert Buckingham, University of Cincinnati

Title: Asymptotics of rational Painleve II solutions
Abstract: The nonhomogenous Painleve II equation has exactly one rational solution for specific values of the nonhomogeneity parameter. Applications of these functions have very recently been discovered to fluid vortices, theoretical physics, and nonlinear wave equations. Clarkson and Mansfield observed numerically that the zeros (or poles) of these rational solutions appear to have a remarkably regular triangular structure. We prove that, as the nonhomogeneity parameter tends to infinity, the scaled zeros (or poles) fill out a certain curvilinear triangular region in the complex plane. We also discuss progress on computing the leading-order asymptotic behavior of the rational solutions inside, outside, and at the edge of this root region. This is joint work with Peter Miller.

(**3**). Ko-Shin Chen, Indiana University

Title: *Ginzburg-Landau vortices on 2-D surfaces*
Abstract: We investigate Ginzburg-Landau energy on compact simply connected 2-manifold. We will show that a critical point is unstable if it has a vortex at a location where the Gauss curvature is positive. Furthermore, for a surface of revolution, any critical point having vortices is unstable regardless of the Gauss curvature at vortex locations. Next, we will discuss the Ginzburg-Landau heat flow on a surface of revolution with boundary. By an extra geometric assumption, we will show that all vortices of the solution disappear after a finite time.

(**4**). Lei Z. Cheng, Purdue University

Title: *Chervon Structures In Liquid Crystal Films*
Abstract: In this presentation, I will begin with a brief mathematical description of liquid crystal phases. They are understood as the intermediate states between liquid and crystalline. Sharing the properties of both, liquid crystals are now widely used in display devices. Liquid crystal molecules self-organize into optimal packing arrangements where the energy is mini- mized. At high temperatures a liquid crystal is in the isotropic phase where the molecules orientations are random as in a common fluid. As the temperature is reduced, the liquid crystal enters different smectic phases where the molecular centers of mass also align, caus- ing the molecules to organize into layers. In particular, physicists have been interested in surface-stabilized cells where the liquid crystals are confined between close glassplates with fixed boundary conditions. Hence, the natural structure is the so-called bookshelf structure with uniform layers perpendicular to the cell plane. Interesting phenomenon occurs when the temperature is further reduced. The bookshelf structure deforms into V-shaped layers called a chevron structure. It changes the pathway of the incident light and therefore distorts the images. This structure constitutes one of the most severe obstacles towards viable LCD devices. Physicists have done a lot of experiments to try to understand this

structure and control the technical difficulties resulting from it. In this talk, I would like to explore this feature analytically from a mathematical point of view.

(**5**). Professor Hi Jun Choe, Yonsei University, South Korea/ University of Kentucky

Title: *Maximum modulus estimates for the solution of the Navier-Stokes equations*
Abstract: A maximum modulus estimate for the nonstationary Stokes equations in C^2 domain is found. The singular part and regular part of Poisson kernel are analyzed. The singular part consists of the gradient of single layer potential and the gradient of composite potential de ned on only normal component of the bound- ary data. Furthermore, the normal velocity near the boundary is bounded if the boundary data is bounded. If the normal component of the boundary data is Dini- continuous and the tangential component of the boundary data is bounded, then the maximum modulus of velocity is bounded in whole domain.

(**6**). Francis Chung, University of Chicago

Title: *A Carleman Estimate for a Partial Data Inverse Problem*
Abstract: I will describe a result for a particular partial data inverse problem for the magnetic Schrodinger operator. I will explain why this result depends on a particular Carleman estimate for that operator, and try to give an idea of how the Carleman estimate can be proved.

(**7**). Sean Colbert, Purdue University

Title: *Analysis of a Ginzburg-Landau Type Energy Model for Smectic C* Liquid Crystals with Defects*
Abstract: This work investigates properties of a smectic C* liquid crystal film containing defects which give rise to distinctive spiral patters in its texture. The phenomena are described by a Ginzburg-Landau type model and the investigation provides a detailed analysis of minimal energy configurations for the film's director field. This investigation demonstrates that a limiting location for the defects (vortices) so as to minimize the energy exists. It is proves that if the degree of the boundary data is positive then the vortices each have degree +1 and that they are located away from the boundary. Through this investigation, a renormalized energy function that depends on the vortices, the boundary values, and the field's patter inside the domain is constructed. It is proved that the limit of the energy functional minus the sum of the energy around the vortices, as the G-L parameter epsilon tends to zero, is equal to this renormalized energy.

(**8**). Gregory Drugan, University of Washington, Seattle

Title: *Self-Shrinking Solutions of Mean Curvature Flow with Rotational Symmetry*
Abstract: In this talk, we will use comparison arguments to study self-shrinking solutions of mean curvature flow with rotational symmetry. We will show that an embedded S^2 self-shrinker in R^3 with rotational symmetry about a line through the origin must be mean convex.

(**9**). Professor Larry Harris, University of Kentucky

Title: *Markovs Theorem for Derivatives of Multivariate Polynomials*
Abstract: We sketch an independent proof of the following.
Theorem. Let X and Y be real normed linear spaces and let $P : X \to Y$ be a polynomial of degree at most m satisfying $\|P(x)\| \leq 1$ for all $x \in X$ with $\|x\| \leq 1$. Then

$$\|\hat{D}^k P(x)\| \leq T_m^{(k)}(1) \text{ for all } x \in X \text{ with } \|x\| \leq 1 \text{and}$$

$$\|\hat{D}^k P(x)\| \leq T_m^{(k)}(\|x\|) \text{ forall } x \in X \text{ with } \|x\| \geq 1.$$

Here $\hat{D}^k P(x)$ denotes the kth order directional derivative of P at x and T_m denotes the Chebyshev polynomial of degree m. We first show that it is sufficient to show just the second inequality when $X = \mathbb{R}^2$ with the max norm, $Y = \mathbb{R}$ and $x = (r, r)$ where r is real. Then we obtain bivariate Lagrange interpolation at nodes that are pairs of Chebyshev points and examine the signs of the directional derivatives of the Lagrange polynomials.

(**10**). Tao Huang, University of Kentucky

Title: *Some new results on the uniqueness of heat flow of harmonic maps and nematic liquid crystal flows*
Abstract: We establish the uniqueness of heat flow of harmonic maps that have sufficiently small renormalized energies. As corollaries, we obtain (i) the uniqueness for heat flow of harmonic maps whose gradients belong to $L_t^p L_x^q$ for $q > n$ and (p, q) satisfying Serrin's condition, and (ii) the uniqueness for hydrodynamic flow (u, d) of nematic liquid crystals, with $(u, \nabla d)$ satisfying Serrin's condition.

(**11**). Zhuomin Liu, University of Pittsburgh

Title: *The Liouville Theorem on Conformal Mappings*
Abstract: The celebrated Liouville Theorem from 1850 states that in dimension $n \geq 3$, the only conformal maps are Mo bious transforms. Liouvilles proof, as well as many subsequent proofs, required the map- pings to be diffeomorphisms of class C^3. However, since C^1 regularity is sufficient to define conformal maps, one may inquire whether the Liouville theorem remains true under that, or even weaker conditions, e.g. Sobolev functions. The reduction from C^3 regularity turned out to be very difficult. It this talk we will discuss the development of the Liouville theorem under weaker and weaker regularity assumptions, including re- sults of Gerhing, Reshnetyak, Bojarski and Iwaniec, Iwaniec and Martin on $W^{1,n}$ conformal mappings. Furthermore, Iwanice and Martin proved loc that in even dimensions $n \geq 4, W^{1,n/2}$ conformal mappings are Mo bious loc transforms and they conjectured that it should also be true in odd di- mensions. We also discuss a proof of the Liouville Theorem $f \in W_{loc}^{1,1}$ in dimension $n \geq 3$ under one additional assumption that the norm of the first order derivative $|Df|$ satisfies $|Df|^p \in W_{loc}^{1,2}$ for $p \geq (n-2)/4$. Meanwhile, we show that the Iwaniec- Martin conjecture can be reduced to a conjecture about the Caccioppoli type estimate.

(**12**). Professor Andrew Lorent, University of Cincinnati

Title: *A brief survey of the Aviles Giga functional*
Abstract: The Aviles Giga functional is a second order functional that is the most natural generalization of the Modica Mortola functional. It serves as a model of blistering and in certain regimes liquid crystal, a closely related functional models thin magnetized films. We will briefly describe and motivate the functional, explain the main open problems and the progress made on them.

(**13**). Professor Leonardo Marazzi, University of Kentucky

Title: *Generic properties of eigenvalues and resonances for compact metrics on surfaces with cusps*
Abstract: I will discuss how simplicity of eigenvalues, the absence of embedded eigenvalues and infinitely many resonances are generic properties for compact metrics on surfaces with cusps. I will also discuss open problems related to this question. This is joint work with P. Hislop and P. Perry.

(**14**). Dr Tuoc Phan. , University of Tennessee, Knoxville

Title: *Navier-Stokes equations in critical spaces: existence and stability of steady state solutions*
Abstract: In this talk, we discuss our recent results on the uniqueness existence of solutions to the stationary Navier-Stokes equations with small singular external forces belonging to a functional space introduced by Mazya and Verbitsky. The stability of the steady state solutions in such spaces is also obtained by a series of sharp estimates for resolvents of a singularly perturbed operator and the corresponding semigroup. The talk is based on the joint work with N. C. Phuc (LSU).

(**15**). Ruipeng Shen, University of Chicago

Title: *Global Well-posedness and Scattering of Defocusing Energy Subcritical Nonlinear Wave Equation in dimension 3 with radial data*
Abstract: I will talk about the defocusing case of the energy subcritical non-linear wave equation in $\mathbb{R}^3$.

$$\begin{cases} \partial_t^2 u - \Delta u + |u|^p u = 0, & (x,t) \in \mathbb{R}^3 \times \mathbb{R}; \\ u|_{t=0} = u_0 \in \dot{H}^s(\mathbb{R}^3); \\ \partial_t u|_{t=0} = u_1 \in \dot{H}^{s-1}(\mathbb{R}^3). \end{cases} \tag{0.1}$$

Here

$$p = \frac{2}{3/2 - s}.$$

We assume the initial data is in the space $\dot{H}^s \times \dot{H}^{s-1}$ and radial. If $s = 1$, the initial data is in the energy space and the scattering results are known. We will deal with the case $15/16 < s < 1$, so that this problem is $\dot{H}^s \times \dot{H}^{s-1}$ critical. We will prove the global well-posedness and scattering of the solution under the additional assumption that the $\dot{H}^s \times \dot{H}^{s-1}$ norm of the solution is uniformly bounded for all time t in the maximal lifespan of the solution.

(**16**). Hung Tran, University of California, Berkeley

Title: *Partial regularity results for a variational problem for nematic liquid crystal*
Abstract: This is a joint work with Craig Evans. We study the partial regularity of minimizers for certain functionals in the calculus of variations, namely the modi ed Landau–de Gennes energy functional in nematic liquid crystal theory introduced by Ball and Majumdar recently.

(**17**). Xiang Xu, Carnegie Mellon University

Title: *Analysis of a hydrodynamic system modeling vesicle and fluid interactions*
Abstract: We study a $3D$ hydrodynamical system modeling the deformation of vesicle membranes in incompressible viscous fluids. We prove the existence/uniqueness of local strong solutions for arbitrary initial data. Then we establish some regularity criteria in terms of the velocity for local smooth solutions. We also study the stability of the system near local minimizers of the elastic bending energy.