The following problem was taken from the Mathematical Sciences Research Institute's newsletter of 2001. A solution will be posted on the Department's web page by early 2003.

The problem Five cards are drawn from a standard deck of cards, Mathematician A looks at the cards and selects four of them to place face up in a row on the table. The fifth card is placed face down. Mathematician A leaves the room and Mathematician B enters and examines the cards on the table, taking particular note of the order in which they appear. After a moment Mathematician B announces the name and suit of the fifth card. How can this be done? (Of course, mathematicians are honorable people and would not resort to using shills in the audience or communicating outside the room.)

A solution

To simplify describing the solution, we will assume the cards are labelled 1 to 13 (thus ace is 1, jack is 11, and so on). With five cards, we know that we will have at least two of the same suit. We let x and y be numbers of two cards which are in the same suit. We also can choose x and y so that $x = y + n \mathrm{mod} 13$ where $n = 1, \ldots 6$. Mathematician A lays the card y face down, and places x as the first card face up. The two mathematicians have agreed on an ordering of the deck so that both mathematicians know an ordering of the three remaining cards. Let us call a the smallest card, b the middle card and c the largest card under this ordering. The mathematicians have also agreed on a correspondence between the integers 1 to 6 and the six permutations of abc. Thus mathematician A can choose a permutation of the three remaining cards to tell Mathematician B the value of n.

When Mathematician B can examine the the last three cards to find the value of n, then add n to the value of the first card to determine the hidden card.