Homogenization with soft inclusions and interior Lipschitz estimates at every scale

Abstract: In this talk, we will discuss recent work establishing interior Lipschitz estimates at the large scale (via a Campanato-type iteration) and at the small scale (via a layer potential argument) for solutions to systems of linear elasticity with ε-periodic coefficients and Dirichlet boundary conditions in domains with periodically placed inclusions of size ε and magnitude δ by establishing H^1-convergence rates for such solutions. In particular, we consider the operator

$$\mathcal{L}_{\varepsilon,\delta^2}(u_{\varepsilon,\delta}) = -\frac{\partial}{\partial x_i}\left[k_{\delta^2}\left(\frac{x}{\varepsilon}\right)a_{ij}^{\alpha\beta}\left(\frac{x}{\varepsilon}\right)\frac{\partial u_{\varepsilon,\delta}^{\beta}}{\partial x_j}\right],$$

where $k_\delta = 1$ in a periodic, connected substrate and $k_\delta = \delta \in [0,1]$ in the complement. In particular, the equation generalizes the setting of perforated domains.

The homogeneous Dirichlet boundary value problem models relatively small elastic deformations of composite materials reinforced with soft inclusions subject to zero external body forces and with a prescribed boundary deformation. We will provide a background to periodic homogenization, present the optimal interior regularity results and convergence rates, and discuss interesting aspects of the proofs.