ORAM 6
University of Kentucky
Abstracts of Talks listed alphabetically

1. Plenary Lectures

Author: Scott Armstrong, Université Paris-Dauphine
Presentation title: A quantitative theory of stochastic homogenization
Presentation abstract: Stochastic homogenization involves the study of solutions of partial differential equations with random coefficients, which are assumed to satisfy a "mixing" condition, for instance, an independence assumption of some sort. One typically wants information about the behavior of the solutions on very large scales, so that the ("microscopic") length scale of the correlations of the random field is comparatively small. In the asymptotic limit, one expects to see that the solutions behave like those of a constant-coefficient, deterministic equation. In this talk, we consider uniformly elliptic equations in divergence form, which has applications to the study of diffusions in random environments and effective properties of composite materials. Our interest is in obtaining quantitative results (e.g., error estimates in homogenization) and to understand the solutions on every length scale down to the microscopic scales. In joint work with Tuomo Kuusi and Jean-Christophe Mourrat, we introduce a new method for analyzing this problem, based on a higher-order regularity theory for equations with random coefficients, which, by a bootstrap argument, accelerates the exponent representing the scaling of the error the all the way to the optimal exponent given by the scaling of the central limit theorem.

Author: Hans Lindblad, Johns Hopkins University
Presentation title: A sharp counter example to local existence for Einstein equations in wave coordinates
Presentation abstract: We are concerned with how regular initial data have to be to ensure local existence for Einstein's equations in wave coordinates. Klainerman-Rodnianski and Smith-Tataru showed that there in general is local existence for data in H^s for $s > 2$. We give example of data in H^2 for which there is no local solution in H^2. This is joint work with Boris Ettinger.

Author: Camil Muscalu, Cornell University
Presentation title: Multiple vector valued inequalities via the helicoidal method
Presentation abstract: The goal of the talk is to describe a new method for proving vector valued inequalities in harmonic analysis, which we like to call "the helicoidal method". Some applications of it will also be discussed. This is recent joint work with Cristina Benea.

Author: Malabika Pramanik, University of British Columbia
Presentation title: Configurations in sets big and small
Presentation abstract: Does a set of positive Lebesgue measure contain an affine copy of your favourite pattern, say a line of specially arranged points, the vertices of a polyhedron or a geometric sequence on a spiral? Would the answer change if the set is Lebesgue-null, but is still large in some quantifiable

sense? Such problems, involving identification of prescribed configurations, have been vigorously pursued both in the discrete and continuous setting, often with spectacular results. Yet many deceptively simple questions remain open. I will survey the literature in this area, emphasizing some of the landmark results that focus on different aspects of the problem.

Author: Monica Visan, University of California, Los Angeles
Presentation title: Symplectic non-squeezing for the cubic nonlinear Schrödinger equation on R^2
Presentation abstract: We prove that the flow of the cubic NLS in two dimensions cannot squeeze a ball in L^2 into a cylinder of lesser radius. This is a PDE analogue of Gromov's non-squeezing theorem for an infinite-dimensional Hamiltonian PDE in infinite volume. This is joint work with R. Killip and X. Zhang.

2. Contributed talks

Author: Chandan Biswas, University of Wisconsin, Madison
Presentation title: Extremals for the convolution operator with a surface measure on the moment curve
Presentation abstract: The operator defined by convolution with the surface measure on the moment curve $(t, t^2, ..., t^d)$ is a bounded operator from L^p to L^q if $(\frac{1}{p}, \frac{1}{q})$ lies on the line segment joining the points $\left(\frac{2}{d+1}, \frac{2(d-1)}{d(d+1)}\right)$ and $\left(1 - \frac{2(d-1)}{d(d+1)}, 1 - \frac{2}{d+1}\right)$. In this article we prove that there exist functions which extremize the associated inequality. This extends work carried out by Michael Christ for the linear operator defined by convolution with surface measure on the paraboloid.

Author: Maxim Gilula, University of Pennsylvania
Presentation title: A version of Van der Corput's lemma in multiple variables, and more
Presentation abstract: I will briefly outline a real analytic proof of a well-known version of Van der Corput's lemma in multiple variables, originally proven by A. N. Varchenko with algebraic-geometric methods. The tools in the proof are much closer in spirit to the ones in modern proofs of Van der Corput's lemma for oscillatory integrals in one dimension; in particular, the method of stationary phase. Finally, by pushing the tools a little more, we will discover a very descriptive asymptotic expansion for the integrals under consideration.

Author: Jasun Gong, Fordham University
Presentation title: Sierpinski-type fractals are differentiably trivial
Presentation abstract: In recent years, there has been growing interest in Rademacher-type theorems on general classes of metric spaces. Roughly speaking, to what extent is a Lipschitz function a.e. "differentiable" on a given metric space, with respect to a given Borel measure? As a concrete example, one may consider non-self-similar versions of the Sierpinski carpet, but treated as metric

spaces, and each equipped with its natural measure. A result of Mackay-Tyson-Wildrick [MTW] characterizes such Rademacher-type theorems: they are valid if and only if the carpet has positive Lebesgue area. This talk treats the complementary cases. In a joint work with E. Durand-Cartagena and J. Jaramillo, we show that Rademacher-type theorems do not hold on non-self-similar Sierpinski carpets of zero area. Our techniques are independent of [MTW]; instead of moduli of curve families and generalised Poincare inequalities, we apply tools from Weaver's theory of "metric derivations" and apply a rigidity result for them.

Author: Shu Gu, University of Kentucky
Presentation title: Homogenization of Stokes systems and uniform regularity estimates
Presentation abstract: In this talk, we study the uniform regularity estimates for a family of Stokes systems with rapidly oscillating periodic coefficients. By using compactness method, we here establish interior Lipschitz estimates for velocity and L^∞ estimates for the pressure as well as a Liouville property for solutions in $\mathbb{R}^d$. We also obtain the boundary $W^{1,p}$ estimates in a bounded C^1 domain for any $1 < p < \infty$. At last, we will discuss the L^2 convergence rates of Dirichlet problem for Stokes system. This is joint work with Prof. Zhongwei Shen.

Author: Sujeewa Hapuarachchi, University of Louiville
Presentation title: Backward heat equation with time dependent variable coefficients
Presentation abstract: The backward heat equation with time dependent variable coefficient is severely ill-posed in the sense of Hadamard, so we need regularization. we use modified-quasi boundary value method to regularize this problem.

Author: Minh Kha, Texas A & M University
Presentation title: Green's function asymptotics of periodic elliptic operators on abelian coverings of compact manifolds
Presentation abstract: Greens function behavior near and at a spectral edge of a periodic operator is one of what was called by M. Birman and T. Suslina threshold properties. I.e., it depends upon the infinitesimal structure of the dispersion relation at the spectral edge. For a "generic" periodic second-order elliptic operator on a co-compact abelian cover, we will discuss the asymptotics at infinity of the Green's functions near and at the spectral gap edge as long as the dispersion relation of the operator has a non-degenerate extremum there. Previously, analogous results have been known for the Euclidean case only. One of the interesting features discovered is that the rank of the deck group plays more important role than the dimension of the manifold.

Author: Nguyen Lam, University of Pittsburgh
Presentation title: Existence and symmetry of maximizers for a family of Caffarelli-Kohn-Nirenberg interpolation inequalities
Presentation abstract: In this talk, we will use a suitable map to investigate the

sharp constants and optimizers for the Caffarelli-Kohn-Nirenberg inequalities for a wide range of parameters. Moreover, we will compute the best constants and the explicit forms of the extremal functions in numerous cases. This is joint work with Guozhen Lu.

Author: Jiaqi Liu, University of Kentucky
Presentation title: Inverse Scattering and Long-Time Asymptotics for the Derivative Nonlinear Schrodinger Equation
Presentation abstract: The Derivative Nonlinear Schrodinger Equation (DNLS) is a nonlinear dispersive wave equation modeling Alfven waves in plasma physics. In this talk I will present a rigorous analysis of the direct and inverse scattering map in some weighted Sobolev spaces for DNLS. We will also compute the long time asymptotics of the solution. This is a joint work with P. Perry and C. Sulem.

Author: Joseph Lindgren, University of Kentucky
Presentation title: Orbital Stability of Soliton Solutions to Cubic NLSP Presentation abstract: For the cubic nonlinear Schrödinger equation in one dimension there exist equilibrium solutions which are solitary waves. Addition of a potential V changes the dynamics, but for small enough $||V||_{L^\infty}$ we can still obtain stability (and approximately Newtonian motion of the solitary wave's center of mass) for soliton-like solutions up to a finite time that depends on the size and scale of the potential V. Our method is an adaptation of the Lyapunov method employed by Michael Weinstein and others.

Author: Xin Yang Lu, McGill University
Presentation title: Grain Boundary Characteristic Distribution
Presentation abstract: Abstract: A model for evolution of policrystalline materials, proposed by Mullins in the 1950s, is the curvature driven evolution. Predicting the evolution of such systems is desirable. The theory of Grain Boundary Characteristic Distribution (GBCD) was proposed by Kinderlehrer et al. as predictive theory for grain network evolutions. Its mathematical formulation is heavily reliant on optimal transport theory. In this talk we present a rigorous derivation of the GBCD theory in 2D. Joint work with David Kinderlehrer.

Author: Lukas Maly, University of Cincinnati
Presentation title: Trace and extension theorems for BV and Sobolev functions in metric spaces
Presentation abstract: In the general Dirichlet problem, one starts with a domain, prescribes boundary values, and looks at the set of functions on the interior of the domain whose trace on the boundary matches the prescribed boundary values. For domains in complete metric spaces endowed with a doubling measure, we investigate the class of functions defined on the boundary that can be extended to functions of some specified regularity on the interior. Under some rather weak requirements on regularity of the boundary, we find a linear extension operator from a Besov class on the boundary to BV class (or to the Newton-Sobolev class $N^{1,p}$) in the domain. This operator can be used

to find BV extensions of L^1 boundary data. Hence, the trace class of $BV(\Omega)$ is $L^1(\partial\Omega)$ provided that the ambient metric space admits a 1-Poincaré inequality. We will also look into analogous questions for domains with a thick boundary (i.e., with a boundary of Hausdorff co-dimension less than 1). This is a joint work with N. Shanmugalingam and M. Snipes.

Author: Dionyssios Mantzavinos, State University of New York, Buffalo
Presentation title: A new approach to initial-boundary value problems for nonlinear dispersive PDEs
Presentation abstract: Nonlinear dispersive partial differential equations have been a topic of great interest in numerous areas of pure and applied mathematics. In particular, a plethora of impressive results on the initial value problem of these equations have been discovered over the past 60 years using techniques from harmonic analysis, geometry, integrable systems and other fields. The study of initial-boundary value problems for such PDEs, however, remains mostly unexplored and the research efforts towards their understanding are rather sporadic. In this talk, a new approach for studying the well-posedness in Sobolev spaces of such problems is presented. This approach applies to evolution equations of arbitrary spatial order and, in particular, to the celebrated nonlinear Schrodinger and Korteweg-de Vries equations, and relies on deriving appropriate harmonic analysis estimates in the complex Fourier plane.

Author: Chris Marx, Oberlin College
Presentation title: Spectral theory of extended Harper's model and a question by Erdös and Szekeres
Presentation abstract: Extended Harpers Model is a model from solid states physics first proposed by D. J. Thouless in context with the quantum Hall effect. It captures how electrons in a 2d crystal respond to the presence of an external magnetic field. The model allows for a wide range of lattice geometries (parametrized by three coupling parameters) by permitting electrons to hop to both nearest and next nearest neighboring (NNN) lattice sites. In the physics literature the model received extensive attention during the last twenty years, however most of the analysis was limited to numerics; even from a heuristic point of view, a complete picture of the spectral properties of the model that covers all of the parameter space had so far been missing. In this talk we present a complete solution of the spectral theory of the model, which in particular addresses the physically most interesting "self-dual regime in the space of coupling constants. Most notably, we prove that in the entire interior of this regime, the model exhibits a collapse from purely absolutely continuous spectrum to purely singular continuous spectrum which is driven by the symmetry of the NNN. The analysis requires some rather delicate number theoretic estimates, which ultimately required the solution of a problem posed by Erd os and Szekeres. The work is joint with Artur Avila and Svetlana Jitomirskaya.

Author: Rajinder Mavi, Michigan State University
Presentation title: Anderson localization in the Holstein model with a disordered field

Presentation abstract: The phenomenon of Anderson localization is considered physically correct in single and many body disordered systems. By now the mathematical characterization of localization in the single body case is well developed. On the other hand, although significant progress has been made in the field of many body localization, much of the current understanding remains in the form of conjecture. It is therefore instructive to consider relatively simple many body models which are nonetheless physically reasonable. We therefore consider the Holstein model with a randomly disordered potential. The model itself is very rich and is worthy of study even in the one dimensional case. We will show, in any dimension d, that for any compact subset of the spectrum the wavepackets are dynamically localized for sufficiently small kinetic term.

Author: Henok Mawi, Howard University
Presentation title: Existence and Regularity of Solutions to Equations with Convex Gradient Constraint
Presentation abstract: In this talk I will discuss the existence, uniqueness and regularity of viscosity solutions of the PDE of the form $\max\{F(D^2u, x) - f(x), H(Du)\} = 0$, where F is fully nonlinear, uniformly elliptic and convex in its first argument, H is convex, f is a given function and u is the unknown. This is joint work with Ryan Hynd.

Author: Oleksandr Misiats, Courant Institute, New York University
Presentation title: Energy bounds and formation of microstructures in alloys
Presentation abstract: We study a variational problem for the energy functional arising in austenite-martensite phase transitions. In this case, the subtle interplay between the elastic and the surface energy terms often leads to the formation of periodic and self-similar microstructures. In my talk I will address the energy scaling law of the minimizing configuration, the spatial distribution of the energy, and further properties of minimizers.

Author: Chad Money, University of Louiville
Presentation title: Chaos in a Wider Context
Presentation abstract: Though originally formulated for cascades, Devaney chaos can be defined in the context of an arbitrary semiflow (where T is any monoid, X any metric space). We will examine some of the topological consequences of that definition, including some very powerful theorems when T is abelian and some introductory results when T is not.

Author: Carlos Paniagua, University of Louisville
Presentation title: A geometric PDE based model for image segmentation
Presentation abstract: A hybrid image segmentation model that is able to process a wide variety of images is introduced. The model takes advantage of global (region) and local (edge) data of the image to be segmented and combines these forces using weights obtained via an asymptotically stable exponential function. The model is inspired from energy minimization problems and leads to a PDE numerical algorithm. Several preliminary examples are provided showing the improvements of the proposed model over two recently proposed

models in the literature. A number of avenues for future research are also discussed.

Author: Pei Pei, Earlham College
Presentation title: Well-posedness and stability of a Mindlin-Timoshenko (MT) plate model with nonlinear damping and sources
Presentation abstract: This research studies well- posedness and long-time behavior of (Reissner)MindlinTimoshenko plate equations. The main feature of the considered model is the interplay between nonlinear boundary viscous damping and nonlinear boundary source terms. The results include Hadamard local well-posedness, global existence, as well as estimates on the uniform energy decay rates.

Author: Guanying Peng, University of Cincinnati
Presentation title: Gamma-convergence for an anisotropic superconductivity model with magnetic fields
Presentation abstract: We analyze minimizers of the Lawrence-Doniach energy for layered superconductors occupying a bounded generalized cylinder, $\Omega \times (0, L)$, in $\mathbb{R}^3$, where Ω is a bounded simply connected smooth domain in $\mathbb{R}^2$. For an applied magnetic field $\vec{H}_{ex} = h_{ex}\vec{e}_3$ that is perpendicular to the layers with $h_{ex} \sim |\ln \epsilon|$ as $\epsilon \to 0$, where ϵ is the reciprocal of the Ginzburg-Landau parameter, we prove compactness results for various physical quantities of energy minimizers, and derive a Gamma-limit of the Lawrence-Doniach energy as ϵ and the interlayer distance s tend to zero, under the additional assumption that the layers are weakly coupled.

Author: Jacob Shapiro, Purdue University
Presentation title: Semiclassical resolvent bounds in dimension two
Presentation abstract: We give an elementary proof of resolvent bounds for semiclassical Schrodinger operators in dimension two. We require mild decay conditions on the potential. The resolvent norm grows exponentially in the inverse semiclassical parameter, but near infinity it grows linearly. This result builds from the works of several authors, including Burq, Cardoso, Datchev and Vodev.

Author: Gareth Speight, University of Cincinnati
Presentation title: A Measure Zero Universal Differentiability Set in the Heisenberg Group
Presentation abstract: The Heisenberg group H is a metric measure space equipped with translations and dilations. Lipschitz functions on H are Pansu di fferentiable almost everywhere, but H admits no bilipschitz embedding into a Euclidean space. We show that there exists a measure zero 'universal di fferentiability set' in H containing a point of Pansu diff erentiability for every real-valued Lipschitz function. The proof adapts techniques from Banach space theory, showing that existence of an 'almost maximal' directional derivative implies Pansu diff erentiability. Joint work with Andrea Pinamonti.

Author: Eric Stachura, Temple University
Presentation title: Boundary Value Problems for the Anistropic Maxwell System in Lipschitz Domains
Presentation abstract: Boundary value problems for the time-dependent, anisotropic Maxwell system are analyzed in a bounded, Lipschitz domain in $\mathbb{R}^3$. The permittivity ε and the permeability μ are parameters which determine the propagation of radiation in a material, and here are assumed to be 3×3 matrices depending on position. Each element $\varepsilon_{ij}(x)$ and $\mu_{ij}(x)$ is assumed to be only bounded and measurable; no differentiability of these parameters is assumed.

Author: Dmitriy Stolyarov, Michigan State University
Presentation title: Bellman function for Ornstein non-inequalities
Presentation abstract: I will speak about Bellman function approach to Ornstein non-inequalities for homogeneous differential operators. The Bellman method allows to restate the analytic problem in terms of convex geometry. It also provides generalizations to the case of vector functions and the case of anisotropic homogeneity.

Author: Feride Tiglay, Ohio State University, Newark
Presentation title: The periodic Cauchy problem for the Hunter-Saxton equation in Besov spaces
Presentation abstract: We show that the Hunter-Saxton equation is well-posed (existence, uniqueness and continuous dependence) in Besov spaces. Furthermore we construct an example which demonstrates that the continuity of the data-to-solution map is not uniformly continuous. These are improvements upon earlier results in Sobolev spaces.

Author: Ebru Toprak, University of Illinois
Presentation title: Dispersive estimate for linear Schrödinger equation with real potentials in four dimensions
Presentation abstract: We study the four dimensional Schrödinger operator, $H = -\Delta + V$, in the weighted $L^1(R^2) \to L^\infty(R^2)$ setting when there is obstruction at zero. We are interested $P_{ac}e^itH$ where P_{ac} is the projection operator onto the absolutely continuous spectrum of H. If there is a zero energy obstruction, we establish the low-energy expansion $e^{itH}\chi(H)P_{ac}(H) = O(1/(\log t))A_0 + O(1/t)A_1 + O((t\log t)^{-1})A_2 + O(t^{-1}(\log t)^{-2})A_3$. Here $A_0, A_1 : L^1(R^n) \to L^\infty(R^n)$, while A_2, A_3 are operators between logarithmically weighted spaces, with A_0, A_1, A_2 finite rank operators, and all of the operators are independent of time. We further show that similar expansions are valid for the solution operators to Klein-Gordon and wave equations. Finally, we show that under certain orthogonality conditions, if there is a zero energy eigenvalue one can recover the bound $\|e^{itH}P_{ac}(H)\| \leq |t|^{-2}$ as an operator from $L^1 \to L^\infty$.

Author: Daniel Wang, Sam Houston State University
Presentation title: Variable A_p weights
Presentation abstract: The variable L^p spaces allow the exponent p to be a function with values between 1 and infinity. A sizable theory has been developed on these spaces, such as the boundedness of maximal and singular integral

operators. A natural development is adapting the classical A_p weights to the variable setting. In this talk, we contrast the classical weights and variable weights, and see what properties hold in the variable case. In particular, we have a "reverse factorization" result that allows us to construct variable weights. This is joint work with David Cruz-Uribe (University of Alabama).

Author: Robert Wolf, University of Kentucky
Presentation title: Compactness of Iso-resonant Potentials
Presentation abstract: Bruning considered Schrödinger operators with potential living on a compact manifold that were isospectral. Relating the spectrum to the trace of the heat semi-group, Bruning showed a set isospectral potentials is compact with respect to a Frechet norm for dimensions less than or equal to 3. Similarly, we can consider the resonances of Schrödinger operators with potentials whose support lies inside a ball of radius r, and consider Schrödinger operators that generate the same resonances as some fixed Schrödinger operator. Using the Poisson formula to relate the resonances to the trace of the wave group, we can show that this set of "Iso-resonant" potentials is also compact in a Frechet norm for up to dimension five.

Author: Xiang Xu, Old Dominion University
Presentation title: A stable scheme for a 2D dynamic Q-tensor model
Presentation abstract: We propose an unconditionally stable numerical scheme for a 2D dynamic Q-tensor model of nematic liquid crystals. This dynamic Q-tensor model is considered as a gradient flow generated by the liquid crystal free energy that contains a cubic term, which makes the free energy unbounded from below. By using a stabilizing technique, we obtain an unconditionally stable scheme and establish the unique solvability and convergence of this scheme, which also leads to the well-posedness of the original PDE system.

Author: Kazuo Yamazaki, Washington State University
Presentation title: Global well-posedness and asymptotic behavior of solutions to a reaction-convection-diffusion cholera epidemic model
Presentation abstract: We study the initial boundary value problem of a reaction-convection-diffusion epidemic model for cholera dynamics, namely the susceptible-infected-recovered-susceptible-bacteria (SIRS-B) epidemic PDE model. First, a local well-posedness result relying on the theory of cooperative dynamics systems is obtained. Via a priori estimates making use of the special structure of the system and continuation of local theory argument, we show that in fact this problem is globally well-posed. Secondly, we analyze the local asymptotic stability of the solutions based on the basic reproduction number associated with this model. This is a collaboration work with Xueying Wang.

Author: Jinping Zhuge, University of Kentucky
Presentation title: Convergence Rates in Periodic Homogenization of Systems of Elasticity
Presentation abstract: This talk is concerned with homogenization of systems of linear elasticity with rapidly oscillating periodic coefficients. We establish

sharp convergence rates in L^2 for the mixed boundary value problems with bounded measurable coefficients.

Author: Seyed Zoalroshd, University of South Florida
Presentation title: Some Remark on Single Layer Potentials
Presentation abstract: In this talk, we discuss spectral properties of the single layer and logarithmic operators on the plane. We present some conditions for injectivity of single layer potentials. We provide an isoperimetric inequality for the Schatten norms of logarithmic potentials on quadrilaterals. This can be regarded as a minor improvement and the general case, for ngons with $n \geq 5$, remains open.