

## Computational Commutative Algebra

**Instructor:** Nathan Fieldsteel.

**Location:** CB 343.

**Time:** MWF 11:00 - 11:50.

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**Office Hours:** By appointment.

### Course Description

This will be a course on computational methods in commutative algebra and algebraic geometry, with a particular focus on the computer algebra system `Macaulay2`. In addition to developing proficiency with `Macaulay2`, we will aim to understand the algorithms which it uses to perform explicit computations with polynomial rings and related objects. A tentative list of topics includes but is not limited to : Gröbner bases and their applications, Buchberger’s Algorithm, minimal free resolutions, Betti tables, syzygies, Stanley-Reisner rings, monomial ideals, and the  $F_4$  and  $F_5$  families of algorithms. Depending on time and student interest, we may also discuss other topics at the intersection of mathematics and computation, such as numerical algebraic geometry using `Bertini`, or writing and formally verifying mathematical proofs using `Coq`.

### Prerequisites

No familiarity with programming will be assumed. Familiarity with basic commutative algebra (rings, ideals, modules) will be helpful. Students should have access to a computer.

### Expectations

Optional homework problems will be assigned. Grade will be based on the completion of a computational project on a topic of the student’s choosing.

### Books

- [1] D. A. Cox, J. Little, and D. O’Shea. *Ideals, varieties, and algorithms*. Undergraduate Texts in Mathematics. Springer, Cham, fourth edition, 2015. An introduction to computational algebraic geometry and commutative algebra.
- [2] D. Eisenbud. *The geometry of syzygies*, volume 229 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 2005. A second course in commutative algebra and algebraic geometry.
- [3] D. Eisenbud, D. R. Grayson, M. Stillman, and B. Sturmfels, editors. *Computations in algebraic geometry with Macaulay 2*, volume 8 of *Algorithms and Computation in Mathematics*. Springer-Verlag, Berlin, 2002.
- [4] H. Schenck. *Computational algebraic geometry*, volume 58 of *London Mathematical Society Student Texts*. Cambridge University Press, Cambridge, 2003.