MATH 765

Topics In Algebra: Combinatorial Commutative Algebra

Lecture: M/W/F, 9:00 - 9:50am, Whitehall Classroom Building 343

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Office Hours:	T/Th, 1:00-2:00pm

Text: *Combinatorial Commutative Algebra*, Miller, Sturmfels, ISBN-13: 978-0387237077

Official Course Description: The goal of this course is to introduce the basics of combinatorial commutative algebra. Familiar discrete structures such as graphs, simplicial complexes, poly-hedra, and various types of counting functions live a second life as defining and organization data for algebraic objects. The special classes of ideals and rings which can be built from these objects serve as a rich source of examples for al- gebraists, and help to pioneer new techniques by revealing surprising connections between combinatorics and algebra. Our selection of topics will include the combinatorial aspects of monomial and binomial ideals, determinantal rings, Hilbert series and Hilbert functions, and free resolutions. We will also explore the role symmetry plays in introducing combinatorics and computation into commutative algebra.

Grade breakdown and Class Attendance: Your grade will be based on a short expository paper and an in-class presentation, both occuring at the end of the semester, and class participation. One source of class participation will be discussion of regularly assigned homework exercises.

If you are aware of an impending absence prior to class, please notify me. Reasons for excused absences include: (a) serious illness, (b) illness or death of family member, (c) University-related trips, (d) major religious holidays, and (e) other circumstances found to fit reasonable cause for nonattendance by the professor. Please see the information available on the website of the Ombud. Students may be asked for "appropriate verification" of an absence in order to be excused.

Disability Services: If you have a documented disability that requires academic accommodations, please see your instructor as soon as possible. In order to receive accommodations in this course, you must provide your instructor with a Letter of Accommodation from the Disability Resource Center. The Disability Resource Center coordinates campus disability services available to students with disabilities. It is located on the corner of Rose Street and Huguelet Drive in the Multidisciplinary Science Building, Suite 407. You can reach them via phone at (859) 257-2754 and via email at drc@uky.edu.

Academic Integrity: Students are expected to adhere to University policy on cheating and plagiarism in all courses. The minimum penalty for a first offense is a zero on the assignment on which the offense occurred. If the offense is considered severe or the student has other academic offenses on their record, more serious penalties, up to suspension from the University may be imposed. Each student is advised to become familiar with the various forms of academic dishonesty as explained in the Code of Student Rights and Responsibilities, which can be found at the website of the Ombud.

Timeline: Relevant book sections are in parentheses.

8/28: Overview
8/30: Ideals, Varieties
9/1: Algorithms

9/4: No class, Labor Day
9/6: Monomial Ideals (1.1,1.2)
9/8: Square-Free Monomial Ideals, Simplicial Complexes (1.3, 1.4)

9/11: Betti Numbers (1.5)
9/13: Group Actions on Ideals (2)
9/15: Borel Fixed Ideals (2)

9/18: Cellular Resolutions (4.1, 4.2)
9/20: Cellular Resolutions (4.3, 4.4)
9/22: Cellular Resolutions (4.5)

9/25: Alexander Duality (5.1, 5.2)
9/27: Duality for Resolutions (5.3, 5.4)
9/29: Projective Dimension and Regularity (5.5)

10/2: Genericity (6.1, 6.2)
10/4: Genericity via Deformation (6.3)
10/6: Bounds and Cogenericity (6.4, 6.5)

10/9: Semigroups, Lattice Ideals (7.1)
10/11: Polyhedral Cones (7.2)
10/13: Hilbert Bases (7.3)

10/16: Initial Ideals of Lattice Ideals (7.4)
10/18: Initial Ideals of Lattice Ideals
10/20: Initial Ideals of Lattice Ideals

10/23: Muligraded Algebras (8.1)
10/25: K polynomials (8.2)
10/27: Multidegrees (8.3 - 8.5)

10/30: Betti Numbers of Lattice Ideals (9.1)
11/1: Laurant Monomial Modules (9.2)
11/3: Resolution of Lattice Ideals (9.3, 9.4)

11/6: Toric Varieties (10.1)
11/8: Fans and Toric Varieties
11/10: Quotients and Toric Varieties (10.3, 10.4)

11/13: Flag Varieties (14.1)
11/15: Khovanskii Bases
11/17: Gel'fand-Zetlin Polytopes (14.4)

11/20 - 11/22: Thanksgiving

11/27-12/8: Class Presentations

MA/PHY506 Fall 2017
Problem Set 9
DUE: 8 December 2017

1. Arfken, Chapter 8, pages 387, problem 8.2.1.

2. Arfken, Chapter 8, page 394: problem 8.3.2.

3. Show that the linear operator $L = -d^2/dx^2$ on $L^2([0, 2\pi])$ is hermitian on the functions that satisfy periodic boundary conditions: $y(0) = y(2\pi)$ and $y'(0) = y'(2\pi)$, and that are twice differentiable. That is, for any two such functions

$$\int_0^{2\pi} \overline{f}(x)(Lg)(x)\ dx = \int_0^{2\pi} \overline{Lf}(x)g(x)\ dx.$$

   Find the normalized eigenfunctions of L, that is, functions satisfying $Lf = \lambda f$, with these properties, and the corresponding eigenfunctions. Check that the eigenfunctions are orthogonal.

4. Consider the nonhomogeneous BVP: $y'' = x(x-2\pi)$ on $[0, \pi]$. Expand y in the eigenfunctions of the related Sturm-Liouville problem $Ly = -y'' = \lambda y$ with DBC at 0 and π. Expand $h(x) = x(x - 2\pi)$ in the eigenfunctions of this Sturm-Liouville problem. Find a formal series solution for y.

5. Find the Fourier series for a square wave:

$$f(x) = \begin{cases} h/2 & 0 < x < \pi \\ -h/2 & -\pi < x < 0 \end{cases}$$

   What is the value of the series at $x = -\pi, 0, \pi$? Is this reasonable?