

MA 714: Topics in Discrete Mathematics Combinatorial Applications of Local Cohomology¹ Spring 2012

1. GENERAL INFORMATION

Prof. Benjamin Braun

Course Webpage:

<http://www.ms.uky.edu/%7ebraun/>

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Class Time/Location: 1:00-1:50PM, MWF, CB 335

Office Location/Hours: 831 POT, 9AM MW and 10AM F, or by appointment

2. COURSE DESCRIPTION

Local cohomology is an algebraic tool used as a measure of complexity for modules over local or graded rings. It has proven quite useful in combinatorics, in large part due to its use in characterizing Cohen-Macaulay modules. This course will survey combinatorial tools and applications inspired by and/or derived from local cohomology. We will focus on a partial selection of the following results (specific topics will be chosen based on student and instructor interest):

- Stanley's proof of the Upper Bound Theorem for simplicial spheres
- Ehrhart-Macdonald reciprocity for rational polytopes
- Hochster's theorem regarding Cohen-Macaulay-ness of normal semigroup algebras
- Hochster's formula for the Hilbert series of the local cohomology of Stanley-Reisner rings
- Reisner's characterization of complexes with Cohen-Macaulay Stanley-Reisner rings
- The Eagon-Reiner characterization of Cohen-Macaulay Stanley-Reisner rings via resolutions of Alexander duals

The background material we will discuss will be a proper subset of the following, depending on what is needed during the course:

- (1) Combinatorics
 - Rational generating functions, quasi-polynomials, polyhedra, f - and h -vectors for simplicial polytopes, Dehn-Sommerville relations, the Schützenberger-Kruskal-Katona theorem, P -partitions, Ehrhart theory, shellability
- (2) Commutative Algebra
 - The Hilbert basis theorem, localization, the Noether normalization lemma, depth, Krull dimension, Cohen-Macaulay modules, Stanley-Reisner rings, local cohomology
- (3) Algebraic Topology
 - Simplicial complexes, Euler characteristic, reduced simplicial homology and cohomology, Betti numbers, the Mayer-Vietoris long exact sequence, simplicial Alexander duality

The best prerequisites for this course are mathematical maturity and an adventurous spirit. However, students enrolling in this course should also have prior experience with at least one of the following areas: commutative algebra, algebraic topology, algebraic geometry, polyhedral geometry, and/or enumerative combinatorics. This course will be of interest for students in discrete mathematics, algebra, and topology.

¹I reserve the right to change or amend this syllabus at any time for any reason.

3. TEXTS

- Combinatorics and Commutative Algebra, 2nd edition. R. Stanley
- Enumerative Combinatorics, Vol 1, 2nd edition. R. Stanley.
- Algebraic Combinatorics on Convex Polytopes. T. Hibi.
- Cohen-Macaulay Rings, revised edition. W. Bruns and J. Herzog.
- Combinatorial Commutative Algebra. E. Miller and B. Sturmfels.
- Twenty-Four Hours of Local Cohomology. S. Iyengar, et al.

I'm mainly planning to follow Stanley's book. There is a nice set of notes at <http://www.math.cornell.edu/~eranevo/homepage/FaceRingNotes.pdf> that I will likely use as a reference as well.

If you are primarily an algebraist or topologist, you might prefer the book "Twenty-Four Hours of Local Cohomology" to the "green book" of Stanley. If you already have a copy of one of the books by Miller/Sturmfels or Bruns/Herzog, these should also be sufficient as references for the course.

4. COURSE EXPECTATIONS

- Students are expected to attend all classes. If students need to miss class for a legitimate reason, they are expected to let me know ahead of time.
- There will be a few problem sets assigned through the semester.

5. COURSE GRADES

Your total grade will be based on your regular course attendance and successful completion of problem sets.

6. ACADEMIC INTEGRITY AND CLASSROOM DEMEANOR

All students are expected to follow the academic integrity standards as explained in the University Senate Rules, particularly Chapter 6, found at:

<http://www.uky.edu/USC/New/SenateRulesMain.htm>

Turn off all cell phones, pagers, etc. prior to entering the classroom. *You are not to use your cell phones, pagers, or other electronic devices during class.* An attitude of respect for and civility towards other students in the class and the instructor is expected at all times.

7. CLASSROOM AND LEARNING ACCOMMODATIONS

Any student with a disability who is taking this course and needs classroom or exam accommodations should contact the Disability Resource Center, 257-2754, room 2 Alumni Gym, jkarnes@uky.edu.