MA 671 & 672 - COMPLEX ANALYSIS

J. E. BRENNAN

Complex analysis, or the study of functions of a complex variable, arose from a dilemma that confronted Italian mathematicians during the Renaissance. It had been known for over three millenia that the formula for the roots of a quadratic polynomial sometimes leads to fictitious expressions involving $\sqrt{-1}$. In all such cases, however, it was generally agreed that the polynomial in question simply had no roots and no further consideration was given to these apparent oddities. But, with the publication of the *Ars Magna* by Cardano in 1545 the situation changed dramatically. Here, for the first time a solution of the general cubic equation

$$x^3 + px + q = 0$$

was explicitly given in terms of the coefficients p and q. Moreover, it soon became clear that, even for cubics having three real roots, the solution could not be obtained by Cardano's method without first introducing the mysterious $\sqrt{-1}$. Later in the 19th century Otto Hölder would prove that this phenomenon is in the very nature of things, but from 1545 onward it became increasingly clear that complex numbers were not just a curiosity and had to be taken seriously.

As it is presently understood, complex analysis lies at the very heart of mathematics. It is here that analysis, geometry, number theory and physics all come together. This course is intended to provide a glimpse of the manner in which that interaction takes place. Along the way we shall consider such seemingly diverse topics as:

(1) the prime number theorem: $\pi(n) \sim n/\log n$

(2) normal families and conformal mapping

(3) polynomial and rational approximation

(4) potential theory and partial differential equations

(5) Riemann surfaces and algebraic functions

In addition to providing background for work in analysis, the course should also be of interest to those intending to concentrate their efforts in algebra, algebraic geometry, number theory, topology and physics.

There are essentially three points of view from which to begin the study of complex analysis, due principally to Cauchy, Weierstrass, and Riemann. These approaches emphasize integral formulas, power series and conformal mapping, respectively. I have chosen to adopt the approach of Weierstrass and to emphasize from the outset the notion of a power series. From that point of view many of the central ideas and theorems of complex analysis arise in a rather natural way, and are quite suggestive of what might be true in a much wider context.

The following texts have been placed on reserve in the science library:

(1) L.V. Ahlfors, *Complex Analysis*, 3rd Edition

(2) J.B. Conway, *Functions of One Complex Variable*, vols. 1 & 2.

(3) T.J. Gamelin, *Complex Analysis*

(4) R. E. Greene & S. Krantz, *Function Theory of One Complex Variable*

(5) E, Hille, *Analytic Function Theory*, vols. 1 & 2.

(6) K. Knopp, *Theory of Functions*

(7) T. Needham, *Visual Complex Analysis*

(8) W. Rudin, *Real and Complex Analysis*

(9) D. Sarason, *Complex Function Theory*

GRADING

Grades will be determined on the basis of a midterm examination, a final examination and homework tentatively scheduled as follows:

(1) Midterm examination	Wednesday, February 17
(2) Final examination	Monday, May 5, 10:00-12:00
(3) Homework	Assigned weekly and due on Friday

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF KENTUCKY, LEXINGTON, KY 40506
E-mail address: brennan@ms.uky.edu