MA 213

Text: *Calculus* 5th ed., by James Stewart, Brooks/Cole, New York 2003

Prerequisite: MA 114 (second semester calculus) or equivalent

Background: This course is intended to be an introduction to the calculus of higher dimensions. Starting with the coordinatization of three dimensional space via Cartesian coordinates, the algebraization of geometry continues with the introduction of vectors - directed line segments. Geometric operations on line segments may be interpreted as algebraic operations on vectors to give an algebra for vectors in both the plane and three dimensional spaces. Not only does this include addition and multiplication by scalar quantities, it also provides two different types of multiplication - the dot and cross products. These products are geometrically defined and we spend some time applying vector algebra to the geometry of three dimensions. The applications of this are to motion of a body in space and the geometry of the motion. If time permits one shows that the geometry of curves in space is characterized by the Frenet- Serret formulas. More importantly, there is the extension to functions of several variables. Here the analogs of many of the formulae from the calculus of one variable are more complicated. When considering higher derivatives we find that there are phenomena that do not occur in the calculus of functions of a single variable. One of the consequences of this is the existence of critical points of saddle type which make the application of several variable calculus to optimization problems far more involved than in the single variable case. Applications include the method of Lagrange multipliers for constrained problems. The integral calculus also generalizes to functions of several variables and this is developed in the fourth part of the course. Applications include moments of inertia as well as calculation of masses, centers of mass and several integral identities that come from the application of the change of variables formula for integrals of functions of several variables. This involves the use of the Jacobian and the connection with linear algebra is developed in a self contained fashion for integrals in both two and three dimensions. The geometry of the regions of integration is a far more crucial ingredient in these calculations.

As with the vector algebra, there are several possible derivatives for a vector field - the curl and the divergence corresponding to the cross and dot products respectively. For scalar functions the corresponding derivative is the gradient. There are surprising relations between these - the curl of a gradient is always zero as is the divergence of a curl. These can also be expressed as identities involving integrals of vector fields over one, two and three dimensional regions, respectively. The mathematical expression of these identities are Stokes, Gauss' and the fundamental theorem for line integrals. Physical applications of these appear as identities involving total charge, angular momentum in electromagnetism and mechanics and conservative vector fields in all areas of physics and mechanics respectively.

The following topics from the text are expected to be covered:

Chapt. 13 Vectors and the Geometry of Space

Chapt. 14 Vector Functions

Chapt. 15 Partial Derivatives

Chapt. 16 Multiple Integrals

Chapt. 17 Sections ≥ 1-5 Vector Calculus

At the end of this course students should be able to:

Manipulate and analyze functions of several independent variables using the differential calculus of several independent variables

Interpret geometrical properties of functions of several variables in terms of algebraic properties of their defining formulae in Cartesian and other coordinates.

Use the calculus of several variables to set up and solve optimization problems involving functions of several variables, including interpretation of the critical points of the problem and the method of Lagrange multipliers for constrained problems

Use the integral calculus of functions of two and three variables to solve physical and geometrical problems in Cartesian, cylindrical, spherical or other coordinate systems.

Analyze situations from physics and mechanics involving the differential and integral calculus of vector fields, including determination of scalar potentials for conservative vector fields and interpretation of integrals of vector fields over surfaces and three