Hermann Grassmann

1809 to 1877

Grassmann is chiefly remembered for his development of a general calculus for vectors.

Find out more at:
http://www-history.mcs.st-andrews.ac.uk/history/
Mathematicians/Grassmann.html

MA 213-003/4
Calculus 3
Fall 2008

Instructors:
Lectures Prof. Richard Carey - 003/4
Recitations Mr. Weifeng Zhi - 003/4

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<th>MWF</th>
<th>1100-1150</th>
<th>CB 349</th>
<th>Office POT 965</th>
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<td>Phone 257-3745</td>
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<td>E-MAIL <a href="mailto:carey@ms.uky.edu">carey@ms.uky.edu</a></td>
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<tr>
<th>Section 003</th>
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Prerequisite: MA 114 (second semester calculus) or equivalent

Background: This course is intended to be an introduction to the calculus of higher dimensions. Starting with the coordinatization of three dimensional space via Cartesian coordinates, the algebraization of geometry continues with the introduction of vectors - directed line segments. Geometric operations on line segments may be interpreted as algebraic operations on vectors to give an algebra for vectors in both the plane and three dimensional spaces. Not only does this include addition and multiplication by scalar quantities, it also provides two different types of multiplication - the dot and cross products. These products are geometrically defined and we spend some time applying vector algebra to the geometry of three dimensions. The applications of this are to motion of a body in space and the geometry of the motion. If time permits one shows that the geometry of curves in space is characterized by the Frenet-Serret formulas. More importantly, there is the extension to functions of several variables. Here the analogs of many of the formulae from the calculus of one variable are more complicated. When considering higher derivatives we find that there are phenomena that do not occur in the calculus of functions of a single variable. One of the consequences of this is the existence of critical points of saddle type which make the application of several variable calculus to optimization problems far more involved than in the single variable case. Applications include the method of Lagrange multipliers for constrained problems. The integral calculus also generalizes to functions of several variables and this is developed in the fourth part of the course. Applications include moments of inertia as well as calculation of masses, centers of mass and several integral identities that come from the application of the change of variables formula for integrals of functions of several variables. This involves the use of the Jacobian and the connection with linear algebra is developed in a self contained fashion for integrals in both two and three dimensions. The geometry of the regions of integration is a far more crucial ingredient in these calculations.

As with the vector algebra, there are several possible derivatives for a vector field - the curl and the divergence corresponding to the cross and dot products respectively. For scalar functions the corresponding derivative is the gradient. There are surprising relations between these - the curl of a gradient is always zero as is the divergence of a curl. These can also be expressed as identities involving integrals of vector fields over one, two and three dimensional regions, respectively. The mathematical expression of these identities are Stokes, Gauss' and the fundamental theorem for line integrals. Physical applications of these appear as identities involving total charge, angular momentum in electromagnetism and mechanics and conservative vector fields in all areas of physics and mechanics respectively.

The following topics from the text are expected to be covered:

- Chapt. 13 Vectors and the Geometry of Space
- Chapt. 14 Vector Functions
Chapt. 15 Partial Derivatives

Chapt. 16 Multiple Integrals

Chapt. 17 Sections ≥ 1-5 Vector Calculus

At the end of this course students should be able to:

Manipulate and analyze functions of several independent variables using the differential calculus of several independent variables

Interpret geometrical properties of functions of several variables in terms of algebraic properties of their defining formulae in Cartesian and other coordinates.

Use the calculus of several variables to set up and solve optimization problems involving functions of several variables, including interpretation of the critical points of the problem and the method of Lagrange multipliers for constrained problems

Use the integral calculus of functions of two and three variables to solve physical and geometrical problems in Cartesian, cylindrical, spherical or other coordinate systems.

Analyze situations from physics and mechanics involving the differential and integral calculus of vector fields, including determination of scalar potentials for conservative vector fields and interpretation of integrals of vector fields over surfaces and three

Grading: Your grade will be calculated from the following distribution of points:

Instructor points

Quizzes 100 points

Some quizzes maybe in the form of class presentations.

There will be three exams given in this course. The grading scale for the first two exams – given during the lecture time - will be as follows:

<table>
<thead>
<tr>
<th>Score Range</th>
<th>Grade</th>
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<tbody>
<tr>
<td>90-100</td>
<td>A</td>
</tr>
<tr>
<td>80-89</td>
<td>B</td>
</tr>
<tr>
<td>70-79</td>
<td>C</td>
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<tr>
<td>60-69</td>
<td>D</td>
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<tr>
<td>Below 60</td>
<td>E</td>
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The grading scale for the final exam will be as follows:

<table>
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<th>Score Range</th>
<th>Grade</th>
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<tr>
<td>117-130</td>
<td>A</td>
</tr>
<tr>
<td>104-136</td>
<td>B</td>
</tr>
</tbody>
</table>
91-103       C
78-90         D
below 78      E

Your course score will be the sum of your test scores and the instructor score.

The grading scale for the course will be as follows:

<table>
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<tr>
<th>Cumulative score</th>
<th>Grade</th>
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<tr>
<td>387-430</td>
<td>A</td>
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<tr>
<td>344-386</td>
<td>B</td>
</tr>
<tr>
<td>301-343</td>
<td>C</td>
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<tr>
<td>258-300</td>
<td>D</td>
</tr>
<tr>
<td>below 258</td>
<td>E</td>
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The exams will be curved in the following way. The mean of all students who earn 40% (55% on the final exam) or more on an exam will be computed. Points will be added to the scores so this mean is adjusted to a score of 75 (97.5 on the final). If the mean is 75 or more, no points are added to the scores. You must bring a photo ID to each exam and you may use a calculator on the exams.

Exam, Quiz and Attendance Policy: In order to be fair to all students, dates of quizzes and exams are firm. It is very important to take each exam on schedule. Missed work may be made up only due to illness with medical documentation or for other unusual (documented) circumstances. (See your Student Rights and Responsibilities [http://www.uky.edu/StudentAffairs/Code/]). Students who have university excused absences or who have university-scheduled class conflicts with uniform examinations may arrange with their instructor to take the exam at an alternate time. Work-related conflicts are neither university excused absences or university-scheduled absences. If you miss an exam, you receive a zero. You will be eligible for a make-up only if you present a valid excuse to me before the exam. If you cannot find a reasonable arrangement for a make-up, contact the department DUS Russell Brown. If you miss 4 recitation sections your cumulative score drops by 10%, i.e., from A to B. If you miss 5 recitation sections your cumulative score drops 15%; if you miss 6 recitation sections you lose 20%, e.g., A to C. If you miss 7 or more recitation sections you get an E. This policy begins the week of Jan. 21, 2008.

Excused Absences: S.R. 5.2.4.2 defines the following as acceptable reasons for excused absences:

1. serious illness;
2. illness or death of family member;
3. University-related trips;
4. major religious holidays;
5. other circumstances you find to be "reasonable cause for nonattendance."

Students anticipating an absence for a major religious holiday are responsible for notifying the instructor in writing of anticipated absences due to their observance of such holidays no later than the last day for adding a class. Information regarding dates of major religious holidays may be obtained through the religious liaison, Mr. Jake Karnes (257-2754).
Cheating: Cheating will not be tolerated, and you are responsible for knowing University policy on cheating. The University’s minimum policy for cheating is failure in the course. (Yes, the chair of the department does spend time each semester prosecuting students who thought they’d never get caught!) Cheating can lead to expulsion from the university. For a complete description of University policies on excused absences, cheating, and student responsibilities see UK’s New Academic Offenses Policy can be found at

http://www.chem.uky.edu/research/grossman/acadoffenses/index.htm

For instance, Senate Rule 6.4.11 states:

The minimum penalty for an academic offense is an E in the course in which the offense took place. The repeat option may not be used to remove an E given for an academic offense. If a prior academic offense has been recorded in the Registrar’s Office, the minimum penalty shall be suspension for one semester (or a minimum of four months in those colleges in the Medical Center where the semester system is not in use. Penalties more severe than the minimum may be imposed where warranted by the circumstances.

Our class is a cell phone-free zone. Cell phones must be off & out of sight for the entire class period.

Important Dates

August 27 (Wed.) .................................... First day of classes
September 1 (Mon.) .................................No classes – Labor Day
September 3 (Wed.) ............................... Last day to add a class
September 17 (Wed.) .... Last day to drop a class without a grade
October 3 (Fri.) ...................................... Examination 1
October 20 (Mon.) .....................................Midterm
November 4 (Tues.) .................... No classes – Election Day
November 7 (Fri.) .................... Last day to withdraw from a class
November 26-29 (Wed.-Sat.) ............ No classes-Thanksgiving
November 14 (Fri.) ...............................Examination 2
December 12 (Fri.) ............................. Last day of classes
December 19 (Fri.) ............................. 0100 – 0300 pm Final Examination

Note: There is an official procedure for dropping a course. You haven’t withdrawn if you simply quit attending.