The attached packet consists of two parts. The student packet will be duplicated and copies will be given to the lecturers to distribute to students.

Student packet:

1. Syllabus
2. Calendar
3. Common errors
4. Suggestions on how to study for calculus.
5. Induction handout and written assignments 1 and 2.

Instructor information

1. Outline of TA and lecturer duties.
2. A suggestion for the first recitation.
3. Pretest, solutions and summary statistics.

Russell Brown
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Calculus I *Spring 2007*
MA113

Textbook: The textbook for this course will be *Calculus*, 5th edition, by James Stewart.

Material to be covered: In Calculus I, we will learn about derivatives, integrals and the fundamental theorems of calculus. We begin by introducing the notion of a limit. Limits are essential to defining derivatives and integrals. By the end of the semester students should know precise definitions of the derivative and the integral and the fundamental theorem of calculus which gives the relation between the derivative and the integral. We will illustrate the methods and ideas of calculus by studying several physical and geometric problems. We will study the interpretation of the derivative as velocity or slope of a tangent line, the trajectory of a body falling under the influence of gravity, the interpretation of the integral as area or distance traveled and the use of the integral in computing volumes of familiar solids such as a sphere or a cone.
We will cover most of Chapters 1 to 6 of Stewart. Please see the course calendar for a detailed listing of sections.

Homework: The bulk of homework for this course will be completed using the web-based homework system at `http://www.mathclass.org`. See below for administrative details on using this website.
At this web site, students will find homework assignments which will count towards your grade. The graded assignments are A1–6, B1–7, C1–7 and D1–4.
There are several web homework assignments that will not be counted towards your grade. The review assignments AR, BR, CR and DR are study guides for each exam. All students should complete these review assignments. There are optional assignments, AW, BW, CW, and DW which are intended to provide additional practice on basic skills. The optional assignment A0 is intended to introduce students to the syntax needed to enter mathematical expressions in the web homework system.
The course calendar lists optional homework assignments from the textbook. These are intended for students who feel they need more practice to master a topic.
In addition to the web homework, we will have seven written assignments that will be graded by humans. These written assignments will be graded for mathematical correctness, and for clarity of exposition. Students who wish to receive full credit should write in complete sentences and use mathematical notation correctly.
Your homework and attendance grade is based on three components: the web homework (95 points), the written assignments (70 points) and attendance in lecture (35 points). Your web homework grade is the minimum of 95 and your your percentage score on web homework. You may find this score at `www.mathclass.org` by clicking homework scores on the main page. Each written assignment will be worth 10 points for a total of 70. Lecturers will take attendance 9 times in lecture. Students will earn 5 points for each time they are marked present, up to a maximum of 35. The total number of points is 200 which will be divided by 2 to obtain a homework score of between 0 and 100.

Late homework: No late submissions of web homework will be accepted. If an emergency

or illness takes you away from school, please meet with your lecturer to discuss your situation and ask to be excused from an assignment, if appropriate. If you have a scheduled absence (travel or authorized university absence) you must still submit the web homework by the deadline.
Written assignments are due at the beginning of lecture. If an emergency or unexpected absence prevents you from turning in the assignment, please see your lecturer to request permission to turn in the assignment late. If you have a scheduled absence (travel or authorized university absence) you should arrange to turn in your paper before leaving school. Unexcused and late submissions will be penalized 10% if the paper is turned in late on the due date and an additional 20% for each day that it is late.

Exams: There will be three exams and a final. These exams are scheduled in the evening as indicated in the course calendar. Please be sure that you have these dates free. The final exam will be cumulative, but with an emphasis on the material covered since the third exam.

MA193: In addition, to the 4 hours of credit for MA113, the department offers one additional hour of credit as MA193 on a pass/fail basis. You will pass MA193 if you have 0, 1 or 2 unexcused absences and you pass MA113. If you have three or more unexcused absences or you fail MA 113, you will fail MA193. Your section number for MA193 should equal your section number for MA113. If you drop or change sections of MA113, please make sure to also drop or change sections of MA193.

Grading: Your grade will be based on the activities in the table below.

3 exams	300
Final exam	100
Homework and attendance	100
TOTAL	500

Students need an average of 90% (450 points) for an A, 80% (400 points) for a B, 70% (350 points) for a C and 60% (300 points) for a D. Grades may be curved by adding a few points to each students course total.

Calculators: Students may use a graphing calculator on exams and homework. Students may not use a machine with symbolic manipulation capabilities on exams. Thus, no TI-89's, TI-92's, no HP-48's or laptop computers may be used on exams. Please see the lecturer if you have any questions as to whether a particular machine may be used on a test. We may clear the memory of calculators before or during an examination.

Absences: You should attend class. If you must miss a recitation and are registered for MA193, you must explain your absence to your teaching assistant. Otherwise, your absence will be marked as unexcused and this may lead to failing MA193.
Attendance will be taken in lecture. If you miss lecture, please speak with your lecturer to see if an absence can be excused.

Web page: A web page for this course is at
`http://www.msc.uky.edu/rbrown/courses/ma113.s.07` Any handouts will be available at this address. Solutions to exams and written assignments will be posted at this website.

Cheating: Students are encouraged to work together to understand a problem and develop a solution. However, the solution they submit for credit must be their own work. However, each student should write their final solution independently. Students should not permit others to use their account at `www.mathclass.org`. Copying on exams is not allowed. Students may not use books or notes during examinations.

Web homework: Students who have pre-registered for MA 113 will have an account at `www.mathclass.org`. Your user name for this account is the e-mail address that you have on file with the University. The password will be the characters u$ followed by the last 6 digits of your social security number, *e.g.* u$654321. If you have difficulty logging in, you may visit Mathskeller (CB 065) M–F from 9am–4pm. You may look up your user name/e-mail address by going to `www.mathclass.org`, clicking on the link to login and then the link titled `Don't know which User Name or e-mail to use?`.

Students who registered near the beginning of the semester may not have an account. Their account will be created automatically within one day of registering for the course. Students who are having difficulty with accounts should speak with their instructor or use the help link at `www.mathclass.org`.

Students who choose to drop MA 113 must drop through the registrar's office. Dropping your registration at `www.mathclass.org` will have no effect on your official registration. Students who switch sections of MA 113 during add-drop will have their registration at `www.mathclass.org` updated automatically. When a student changes sections of MA 113 with the registrar's office, the account and record of homework will be automatically transferred to the new section at `www.mathclass.org`.

Web homework problems will be discussed in recitation on Tuesday and Thursday and submitted by 12 midnight on the following Monday. Students should attempt homework as soon as the corresponding material is discussed in lecture. Students who wait till the due date to begin an assignment will likely not complete the work on time.

Each student will have an individual version of the homework. Students should plan to print out their assignment, complete the problems in a notebook, submit their answers and then rework problems or seek assistance for problems that were marked incorrect. Your instructors will want to see the progress you have made in order to provide assistance. In addition, there is a common version of each homework set. The problems from the common version will be discussed in recitation.

If you feel you have worked a problem correctly and WHS marks it incorrect, please contact Russell Brown (by e-mail to `russell.brown@uky.edu` or by submitting the form at `http://www.msc.uky.edu/rbrown/whs/report.html`).

Date	Section, topic, assignments.	Assignments (textbook problems are optional)
10-Jan	Ch. 1 Review of functions	Review, p. 56 #1,2,3,5,6,8-12,16-19
		A0: Calculator syntax (optional)
11-Jan	Pretest, Assignment A1.	A1: Review
12-Jan	Mathematical induction, handout.	
15-Jan	Martlin Luther King, Jr. day, Academic holiday	
16-Jan	Written assignment 1	
17-Jan	2.1 Tangent and velocity problems	§2.1 #1,2,3,5,6,8,9
18-Jan	Assignment A2, practice quiz 1	A2: Tangents and velocity
19-Jan	2.2 Limit of a function	§2.2, #2,4,5,6,9,12,13,15,25,28
22-Jan	2.3 Calculating limits using the limit laws	§2.3 #1,2,11,13,15,17,20,22,28,39,40,49
	12m submission deadline for A1 and A2	
23-Jan	Assignment A3	A3: Limits
24-Jan	2.5 Continuity	§2.5, #1,3,4,5,6,7,9,11,17,21,37
	Written assignment 1 due in class.	
25-Jan	Assignment A4, practice quiz 2	A4: Continuity
26-Jan	2.6 Tangents, velocities and rates of change	§2.6 #1,2,3,5,13,15,17,18,23
29-Jan	3.1 Derivatives	§3.1 #3,4,6,7,9,12,15,16,19,22,25,26
	12m submission deadline for A3 and A4.	
30-Jan	Assignment A5, Written assignment 2	A5: Tangents velocity, rates of change
31-Jan	3.2 The derivative as a function	§3.2 #1,2,4,7,10,12,17,25,36,39,41
	Written assignment 2 due in class	
	Last day to drop	
1-Feb	Assignment A6, practice quiz 3	A6: The derivative
2-Feb	Review	AW: warmup exercises (optional)
5-Feb	Review	
	12m submission deadline for A5 and A6.	
6-Feb	Assignment AR (not graded)	AR: Review for exam 1
	7:30-9:30pm, Exam 1, room TBA.	
7-Feb	3.3 Differentiation formula	§3.3 #5,10,16,18,21,25,28,33,40,44,53,57,58,62
8-Feb	Assignment B1	B1: Differentiation rules
9-Feb	Appendix D, review of trigonometry	
12-Feb	3.5 Derivatives of trigonometric functions	§3.5 #3,6,9,12,18,29,30,35,36,43
	12m submission deadline for B1.	
13-Feb	Assignment B2, practice quiz 4	B2: Review of trigonometry
14-Feb	3.6 The chain rule	§3.6 #1,5,6,7,10,15,16,19,25,28,45,46,55,56
15-Feb	Assignment B3, written assignment 3	B3: Differentiation of trigonometric functions
16-Feb	3.7 Implicit differentiation	§3.7 #3,4,7,10,14,15,26,29,35,39
19-Feb	3.8 Higher derivatives	§3.8 #1-3,11,18,25,26,39,41,44,49,50,53
	12m submission deadline for B2 and B3.	
20-Feb	Assignment B4, practice quiz 5	B4: The chain rule
21-Feb	3.9 Related rates	§3.9 #1,2,4,6-8,10-12,14-17,20-22
	Written assignment 3 due in class.	
22-Feb	Assignment B5, Written assignment 4	B5: Implicit differentiation, higher order derivatives
23-Feb	Related rates, continued	
26-Feb	3.10 Linear approximation	§3.10 #1,3,7,8,13,15,27,31,32,37
	12m submission deadline for B4 and B5.	
27-Feb	Assignment B6.	B6: Related rates
28-Feb	4.1 Maximum and minimum values	§4.1 #1,2,3,4,5,9,11,17,18,23,47,48,52
	Written assignment 4 due in class.	
1-Mar	Assignment B7, practice quiz 6	B7: Linear approximation, Extreme values
2-Mar	Review	BW: Warmup exercises (optional)
5-Mar	Review	
	12m submission deadline for B6 and B7.	
6-Mar	Assignment BR (not graded)	BR: Review for exam 2
	7:30-9:30 pm Exam 2, room TBA	

Date	Section, topic, assignments.	Assignments (textbook problems are optional)
7-Mar	4.2 The mean value theorem	§4.2 #1,3,5-8,15-19,22
8-Mar	Assignment C1	C1: The mean value theorem
9-Mar	4.3 Derivatives and the shape of a graph	§4.3 #1,2,5,6,7-9,11-17,22-26,29,31,33
	Last day to withdraw	
12-16 Mar	Spring break, academic holiday	
19-Mar	4.4 Limits at infinity	§4.4 #1-4,9,11,13,15,17,19,21,23,35,37,39,43,58
	12m submission deadline for C1.	
20-Mar	Assignment C2, practice quiz 7	C2: Derivatives and the shape of a graph
21-Mar	4.5 Summary of curve sketching	§4.5 #3,12,13,17,23,27,31, §4.6 #20,21,26,27
22-Mar	Assignment C3, written assignment 5	C3: Summary of curve sketching
23-Mar	4.7 Optimization problems	§4.7 #2,3,6,7,10,16,19,22,29,32,35,51,52.
26-Mar	4.7 continued	
	12m submission deadline for C2 and C3.	
27-Mar	Assignment C4, practice quiz 8	C4: Optimization
28-Mar	4.9 Newton's method	§4.9 #1,4,5,6,11,27,31,34,35
	Written assignment 5 due in class.	
29-Mar	Assignment C5, written assignment 6.	C5: Newton's method
30-Mar	4.10 Anti-derivatives	§4.10 #1,3,5,7,21,23,25,31,36,37,39,40,53,55,68,7
2-Apr	5.1 Areas and distances	§5.1 #1,3,4,5,11,12,20,22
	12m submission deadline for C4 and C5	
3-Apr	Assignment C6.	C6: Anti-derivatives
4-Apr	5.2 The definite integral	§5.2 #1,7,9,17,19,25,29,30,33-36,39,47-49,55,57
	Written assignment 6 due in class.	
5-Apr	Assignment C7, practice quiz 9	C7: Areas and distances: the definite integral
6-Apr	Review	CW: warmup exercises (optional)
9-Apr	Review	
	12m submission deadline for C6 and C7	
10-Apr	Assignment CR (not graded)	CR: Review for exam 3
	7:30pm-9:30pm, Exam 3, room TBA.	
11-Apr	5.3 The fundamental theorem of calculus	§5.3 #1,7-11,13,19,21,23,25,27,31,33,51,
12-Apr	Assignment D1	D1: The fundamental theorem of calculus
13-Apr	5.4 Indefinite integrals	§5.4 #1,3,17,19,21,23,25,31,33,43,46,48
16-Apr	5.5 Substitution	§5.5 #1,3,9,11,13,15,17,19,21,27,37,39,45,49,57,5
	12m submission deadline for D1	
17-Apr	Assignment D2, written assignment 7	D2: Indefinite integrals, substitution
18-Apr	6.1 Area between curves	§6.1 #1,2,5,7,11,13,21,22,24,45
19-Apr	Assignment D3, practice quiz 10	D3: Area between curves
20-Apr	6.2 Volume	§6.2 #1,3,12,13,14,47,48,49,53
23-Apr	6.2 Volume, continued	
	12m submission deadline D2, D3	
24-Apr	Assignment D4, practice quiz 11	D4: Volumes
25-Apr	Review	
	Written assignment 7 due in class	
26-Apr	Assignment DR	DR: Review for exam 4
27-Apr	Review	DW: Warmup exercises (optional)
	12m submission deadline for D4	
3-May	8:30-10:30pm, Final exam, room TBA	

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COMMON ERRORS.

The following errors are commonly found in mathematics homework and exams. Please try to avoid these mistakes. When grading papers, the three letter abbreviations may be used to indicate these errors.

- EQN. Misuse of the equal sign. When we write the equal sign, $=$, the quantities appearing on either side should be equal. It is common for students to write something like $x^2 = 2x$, when they mean $\frac{d}{dx}x^2 = 2x$.
- ALG. Mistakes in algebra. It is not true that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$. Try $a = b = 1$. It is also not correct cancel b^2 to obtain $\frac{a+b^2}{b^2} = a$.
- EXP. Incomplete explanations. Your goal in working a problem in mathematics is not just to arrive at a correct answer, but to also show that you understand why the answer is true. In general, your explanations should be written out in complete sentences. Occasionally, students will find a value of an integral or limit by using their calculator. Such answers will generally not receive credit for lack of explanation.
- UNI. Missing or incorrect units. In applied problems, one should give the units for the answer. Thus the velocity of a wombat might be 10 meters/second or 1000 centimeters/second, but never 10.
- AXE. Label your axes. When drawing a graph, you should give the coordinate for each axis and label a few tick-marks on each axis.

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Some suggestions on how to study mathematics.

- Go to class.
- Go to class and take notes. The notes you produce will provide a written record of what your instructor believes to be the most important points of the course. Your notes will provide additional examples that will be helpful as you solve problems on homework. Writing notes will help focus your attention on the mathematics being presented in class.
- Go to class when exams and written assignments are returned. Be sure to collect every assignment. Review mistakes in your work so that you will be less likely to repeat the mistake on the next assignment.
- Read the textbook before you go to class. While you may not understand every detail after reading a section in the text, you will be familiar with the main ideas and class will help to deepen your understanding.
- Write careful and complete solutions to each homework problem and keep these solutions in a notebook. We will not, in general, grade your solutions. Your solutions will be useful when you review for exams. Your solutions will be helpful when you seek assistance on a problem and need to explain how you have approached the problem.

  In your work, your goal is not just to arrive at a correct answer but to also to explain your reasoning in order to convince the reader that you understand how to solve the problem.
- Form a study group. After you go to class, work on homework with fellow students. Make sure that you are able to answer every question. After you have answered every question, make sure that you understand why your answers are correct. Discuss your reasoning with your fellow students and see if others have a better approach to solving a problem.
- Attempt homework problems immediately after material is covered in class. Mark problems that you find difficult and look for explanations for these problems. You may obtain assistance in Mathskeller, the Study, in recitation, in office hours of your teaching assistant or lecturer, in your study group, and by sending e-mail to your TA from mathclass.org.

  The Study is located in the third floor of the Commons in the Kirwan-Blanding Dormitory Complex and provides drop-in tutoring in a number of subjects. The

study is open in the evenings from Sunday to Thursday. Visit `http://www.uky.edu/UGS/study` for more information.

The Mathskeller is located in the basement of the Classroom building and is open from 9 to 4 Monday to Friday. Instructors in MA 113 will hold some of their office hours in Mathskeller. For a schedule visit `www.mathskeller.org`. In addition, assistance with `mathclass.org` will be available and students may print out their assignments from `www.mathclass.org` in the Mathskeller.

- Be sure to review carefully every item on the review homework assignments.
- Consider taking a practice exam. Exams from past semesters are available at `http://www.msc.uky.edu/rbrown/courses/exams-113/`. Note that there may be small changes in the syllabus from semester to semester.
- Work problems. Work more problems. Many students choose to work additional problems above the required assignments. The following sources of additional problems are available. 1. The textbook (see the course calendar). 2. The common version of web homework problems. 3. Additional versions of your web homework. Choose your favorite number to use as the version number at `mathclass.org`. 4. The warm-up homework sets at `mathclass.org`.
- Did we mention that you should go to class?

Handout on induction and written assignment 1. MA113
Calculus I Spring 2007

Why study mathematical induction? For many students, mathematical induction is an unfamiliar topic. Nonetheless, this is an important topic and useful in the study of calculus. The study of calculus of calculus involves many new ideas. To study derivatives, we have to look at the slope between pairs of points that are arbitrarily close together. To define the integral, we have to subdivide an interval into n sub-intervals for infinitely many values of n. To fully understand these operations, we have to see why infinitely many statements are true. Mathematical induction is one way to see that infinitely many statements are true.

In mathematics, we engage in deductive reasoning. We make assumptions and deduce conclusions from these assumptions. The induction step in a proof by mathematical induction provides practice in this type of reasoning.

Finally, mathematical induction provides a framework which allows us to understand why many important results in calculus, such as the rule for the derivative of a power, are true.

Summation notation. First, we explain use of $\sum$ for summation or repeated addition. The notation

$$\sum_{k=1}^{n} f(k)$$

means to evaluate the function $f(k)$ at $k = 1, 2, \ldots, n$ and add up the results. In other words:

$$\sum_{k=1}^{n} f(k) = f(1) + f(2) + \ldots + f(n).$$

For example:

$$\sum_{k=1}^{4} k^2 = 1 + 4 + 9 + 16,$$

$$\sum_{k=1}^{n} (2k - 1) = 1 + 3 + 5 + \ldots + 2n - 1,$$

and

$$\sum_{k=3}^{2n} 1 = 2n - 2.$$

The principle of mathematical induction is used to establish the truth of a sequence of statements or formula which depend on a natural number, $n = 1, 2, \ldots$. We will use P_k to stand for a statement which depends on k. For example, P_k might stand for the statement "The number $2k - 1$ is odd." These statements are true for $k = 1, 2, \ldots$

The principle of mathematical induction is:

Principle of mathematical induction. Suppose that P_n is a sequence of statements depending on a natural number $n = 1, 2, \ldots$. If we show that:

- P_1 is true
- For each natural number N: If P_N is true, then P_{N+1} is true.

Then, we may conclude that all the statements P_n are true for $n = 1, 2, \ldots$.

To see why this principle makes sense, suppose that we know P_1 is true, then the second step allows us to conclude P_2 is true. Now that we know P_2 is true, the second step allows us to conclude P_3 is true. If we repeat this $n-1$ times, we conclude that P_n is true.
This principle is useful because it allows us to prove an infinite number of statements are true in just two easy steps! We usually call the first step the *base case* and the second step is called the *induction step.*
Below are several examples to illustrate how to use this principle. The statement P_N that we assume to hold is called the *induction hypothesis.* The key point in the induction step is to show how to use the induction hypothesis, P_N, to deduce P_{N+1}.

Example 1. Show that for $n = 1, 2, 3, \ldots$, the number $n^2 - n$ is even.
Solution. Base case. This is easy. If $n = 1$, then $n^2 - n = 1^2 - 1 = 0$ and 0 is even.
Induction step. We suppose that $N^2 - N$ is even and we want to use this assumption to show that $(N+1)^2 - (N+1)$ is even. We write
$(N+1)^2 - (N+1) = N^2 + 2N + 1 - N - 1 = N^2 - N + 2N$. Now $2N$ is even when N is a whole number and $N^2 - N$ is even by our induction hypothesis. As the sum of two even numbers is again even, we conclude that $(N+1)^2 - (N+1)$ is even. ∎

Example 2. Show that for $n = 1, 2, \ldots$, we have

$$\sum_{j=1}^{n} 2j = n(n+1).$$

Solution Base case. If $n = 1$, then $n(n+1) = 1 \cdot 2 = 2$. Also,

$$\sum_{j=1}^{1} 2j = 2.$$

Thus both sides are equal if $n = 1$.
Induction step. Now suppose that the formula $\sum_{j=1}^{N} 2j = N(N+1)$ is true and consider the sum

$$\sum_{j=1}^{N+1} 2j = \sum_{j=1}^{N} 2j + 2(N+1).$$

On the right-hand side, we have written the last term in the sum separately.

We use our induction hypothesis that $\sum_{j=1}^{N} 2j = N(N+1)$ to conclude that

$$\sum_{j=1}^{N+1} 2j = N(N+1) + 2(N+1).$$

Simplifying this last expression gives

$$N(N+1) + 2(N+1) = N^2 + N + 2N + 2 = N^2 + 3N + 2 = (N+2)(N+1).$$

Since $(N+2)(N+1) = (N+1+1)(N+1)$, we have shown that the formula

$$\sum_{j=1}^{N+1} 2j = (N+1+1)(N+1)$$

is true. This completes the induction step and thus the proof by induction. ∎

Example 3. All horses are the same color.
Solution. We will show by induction that any set of N horses consists of horses of the same color.
The base case is easy. If we have a set with one horse, then all horses in the set are the same color.
We assume as our induction hypothesis that any set of N horses consists of horses of the same color. We take a set of $N+1$ horses, and put one of the horses in the barn for a moment. By our induction hypothesis, the remaining N horses are all of the same color. Now, we put a different horse in the barn. Again, the remaining N horses are all the same color. It follows that the set of $N+1$ horses are all the same color. ∎

Written assignment 1. Work the following three problems related to mathematical induction and hand in your solutions. You will have time some time in recitation to begin working on these problems. Write up your solutions neatly, carefully and in complete sentences.

1. (a) For $n = 1, 2, 3, 4$, compute
$$\sum_{k=1}^{n} (2k-1).$$
Make a guess for the value of this sum for $n = 1, 2, \ldots$.
   (b) Use mathematical induction to prove that your guess is correct.

2. Use the principle of mathematical induction to prove that
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

3. Let a be a fixed number and h a variable. For $n = 1, 2, 3, \ldots$, show that there is polynomial q_n so that

$$(a+h)^n = a^n + na^{n-1}h + h^2 q_n(h).$$

Of course, $q_n(h)$ will also depend on a. Hint: For $n = 1$, the polynomial q_1 is particularly simple. We have $q_1(h) = 0$. For the induction step, write $(a+h)^{N+1} = (a+h)(a+h)^N$.

Additional problems. Below are some additional exercises for you to consider. You will not be able to solve all of these problems at this time. These problems will not be collected.

1. Find the flaw in the proof that all horses are the same color.

2. Let $f_1(x) = x - 2$ and then define f_n for $n = 1, 2, \ldots$ by $f_{n+1}(x) = f_1(f_n(x))$. (It is the principle of mathematical induction which tells us that these two statements suffice to define f_n for all $n = 1, 2, 3, \ldots$.) Use mathematical induction to prove that

$$f_n(x) = x - 2n.$$

3. Show that if $r \neq 1$, we have

$$\sum_{k=0}^{n} r^k = \frac{1 - r^{n+1}}{1 - r}.$$

4. Let P_n be the statement: $n^2 - n$ is an odd integer.

   (a) Show that if P_n is true, then P_{n+1} is true.

   (b) Is P_1 true?

   (c) Is P_n true for any n?

5. Let $f(x) = \sin(2x)$. Prove that for $n = 1, 2, \ldots$,

$$\frac{d^{2n}}{dx^{2n}} f(x) = (-4)^{2n} \sin(2x).$$

6. Let $f(x) = xe^x$. Compute f', f'', and f'''. Guess a formula for the nth derivative,

$$\frac{d^n}{dx^n} f(x).$$

Prove that your guess is right.

7. Prove that

$$\frac{d}{dx}x^n = nx^{n-1}, \qquad n = 1, 2 \ldots.$$

Hint: For the base case $n = 1$, use the definition of the derivative. For the induction step write $x^{n+1} = x \cdot x^n$ and use the product rule.

8. Prove that

$$\frac{d}{dx}\frac{1}{x^n} = \frac{-n}{x^{n+1}}, \qquad n = 1, 2 \ldots.$$

9. Prove that

$$\frac{d^n}{dx^n}x^n = n!, \qquad n = 0, 1, \ldots$$

10. (a) Find a simple formula for

$$\sum_{k=1}^{n}((k+1)^2 - k^2) = 2^2 - 1 + (3^2 - 2^2) + \ldots + n^2 - (n-1)^2 + (n+1)^2 - n^2.$$

(b) Using your answer to part a), find a simple expression for

$$\sum_{k=1}^{n}(2k - 1).$$

To do this you should simplify each summand on the left.

11. Use mathematical induction to prove that

$$\sum_{j=1}^{n} j^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

January 5, 2007

Written assignment on the definition of a derivative. MA113
Calculus I Spring 2007

This written assignment is designed to provide practice in using the definition of the derivative to find the slope of a tangent line. Some of the calculations are sometimes a bit tedious. However, they provide good practice in algebra. More importantly, we should learn the basics (nothing in calculus is more basic than the definition of the derivative) well, before we move on to more elegant approaches.
As always, your work should be written out neatly and carefully. Use complete sentences.
For these exercises, it will be useful to recall that we rewrite the difference of radicals by multiplying and dividing by the conjugate:

$$\sqrt{a}-\sqrt{b}=\frac{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}{\sqrt{a}+\sqrt{b}}=\frac{a-b}{\sqrt{a}+\sqrt{b}}.$$

1. Write the definition of the derivative of a function f at a point x.

2. Use the definition of the derivative to find the derivative of $f(x)=\sqrt{5-3x}$. What is the domain of f? What is the domain of f'?

3. Suppose (x,y) is a point on a circle centered at the origin. Using well-forgotten facts from geometry, find the slope of the tangent line to the circle which passes through (x,y). State clearly the fact from geometry that you use to find the slope.

4. Now consider the unit circle, the circle of radius one which is centered at the origin. The part of the unit circle that lies above the x-axis is given by the graph of the function $g(x)=\sqrt{1-x^2}$. Use the definition of the derivative to find the slope of the tangent line to the unit circle at the point $(x,\sqrt{1-x^2})$.

5. Do your answers to parts 3 and 4 agree?

January 4, 2007

Calculus I *Spring 2007*
MA113

Duties of teaching assistants.

1. In general, a recitation should consist of:
   (a) A short "lecture" to review material and work examples. Examples will generally be drawn from the common homework sets at WHS or the textbook.
   (b) A period of group work devoted to the homework set or written assignment of the day and additional activities as you deem appropriate.
   (c) Practice quizzes as in the course schedule.
2. Make a brief syllabus which gives your name, office hours and contact information. Hand this out on the first meeting.
3. Grade pretests and return to the lecturer to be given to the student the next day. The scores on these tests may help to identify one or two students per section who have registered for calculus but are not prepared. This applies to students who score 0-4.5. Students in the range 5-7.5 should be warned to seek out extra practice on routine skills. Students who score 8 and above are prepared, but should not plan to coast through the semester.
4. Teaching assistants must take attendance in recitation in order to assign MA 193 grades. Accept any reasonable excuse for an excused absence. However, the student must make the effort to explain his or her absence.

   I suggest that we not count absences which occur on or before the last day to add, 17 January 2007.

   It might be useful to prepare a list of student names and take attendance by circulating this list to collect signatures.
5. Teaching assistants will be responsible for recording grades. We will record exam grades and written homework grades at mathclass.org. Be sure to download and/or print a copy of grades in the unlikely event that the servers melt down. Lecturers can access grades through webclass. However, please ask to find out if the lecturer(s) that you are working with would prefer a paper copy of grades.
6. Prepare practice quizzes and distribute a copy of the quiz and solutions to each teaching assistant and lecturer by 3:30pm the day before the quiz is to be given. A schedule will be prepared giving assignments for writing quizzes.
7. Teaching assistants should schedule one hour in Mathskeller and two additional office hours at location that is convenient for you and for your students.

8. Teaching assistants will help with proctoring and grading of exams and will do most of the grading of the written assignments.

9. Written assignments should be graded within one week. No written assignment should be handed in before an exam and returned after an exam.

10. Exams will be graded the night of the exam and the following day. Our goal is to complete our grading assignment by 5pm on the day after the exam.

11. Teaching assistants should provide a report to the coordinator on the scores for each exam. Please submit these reports by e-mail before 2pm on the Thursday following a mid-term exam. For each section, this report should include three items:

    - The number of papers scoring above 30.
    - The mean score of the papers scoring above 30.
    - The number of papers scoring 30 or below.

    Papers scoring 30 or below will be discarded in preparing a course average.

    In addition, each lecturer may want additional information about their section.

    (It may be possible to obtain these averages through mathclass.org web-class. If so, this reporting will not be necessary. The coordinator will keep you posted as he learns how to use mathclass.org.)

12. Weekly instructor meeting will be held at 4pm on Monday in the coffee room. If there is a faculty meeting, the course meeting will be canceled. Our goal is to be brief and informative.

13. If a student complains about a grade, ask that the student write down the question and hand it in with the paper. Bring the paper to your lecturer and to review the grading and make changes if necessary.

14. Answer feedback from within mathclass.org. Please try to answer messages once a day during the week. Our students should not expect responses on Saturday and Sunday.

    Please let Brown know if you run across a problem that appears to be incorrectly set.

15. The new mathclass.org seems to allow instructors to have different due dates for an assignment. Do not take advantage of this feature. If one section has a later due date, then students may join a section with an earlier due date, find the answers and submit these answers for credit.

    If there is a need to adjust a due date, please contact Brown and ask that the due date be changed for all sections. The homework assignments that are due the day before exams cannot be delayed.

16. Prepare grade-sheets and give to lecturer for signature.

Duties of lecturers.

1. Make a brief syllabus which gives your name, office hours and contact information. Hand this out on the first day.

2. Deliver brilliant, insightful lectures exposing the beauty and power of the Calculus, a human endeavor.

3. Write exams.

4. Supervise invigilating and grading of exams. We will take turns as "exam tzar". A schedule will be distributed with exam writing and exam tzar assignments.

5. The exam csar should: Prepare a key with point assignments for problems. Provide Brown with a clean copy of the key to scan and post on the web. Answer questions from graders. Review a few exams from each teaching assistant to make sure that the key is being followed and that following the key is producing sensible grades.

6. Each lecturer will handle appeals on grading in their sections. Report to Brown or exam czar if there appear to be serious, systematic mistakes in grading.

7. Review problem sets and make suggestions for problems to be edited, deleted, or added.

8. Review practice quizzes written by your teaching assistants.

9. Take turns leading weekly meetings.

10. Participate in setting any curves on exams. Our goal is produce a course average by 3pm on Thursday after each midterm exam. Information on the mean will be circulated by e-mail and each instructor will submit their suggestion for a curve. The curve will be the average of the suggested curves.

11. Keep a record of excused absences that caused a student to miss a homework assignment. I recommend that students who are excused from homework have their homework grade computed as an average of the assignments they submit.

12. Work with teaching assistants to administer and grade makeup exams.

13. Visit recitations to help TA's develop as teachers. Follow up if serious problems are identified.

14. Review midterm and final grades and sign grade-sheets.

15. Circulate attendance sheets in lecture.

Duties of coordinator.

1. Take credit for our successes, observe that any failures are the fault of the high schools.
2. Adjust due dates for homework, if needed.
3. Prepare schedules for exam tsars, writing exams, and writing practice quizzes.
4. Edit web homework.
5. Prepare makeup exams.
6. Maintain common web site.

Pretest for Calculus I

On a separate sheet of paper, answer the following questions and show your work. Put your name and section number on your answer sheet. Allow 30 minutes for this pretest. A calculator may not be used in answering these questions.

This pretest is to help you determine if you are ready to begin the study of calculus. Your score on this pretest will not be counted toward your grade in MA113, Calculus I.

If you are not able to answer 8 of these questions correctly, then you may have difficulty with Calculus I. The previous sentence is not equivalent to the statement, "If you are able to answer 8 of these questions correctly, then you will not have difficulty with Calculus I"

1. Write as a single rational expression by obtaining a common denominator.

$$\frac{1}{x+1} - \frac{1}{x-1}.$$

2. If the two shorter sides of a right triangle are of length 2 and 3, what is the length of the hypotenuse?

3. Simplify by cancelling common factors.

$$\frac{\cos x \sin x + \sin^2 x}{\sin^2 x}.$$

4. Factor the numerator and denominator and cancel common factors.

$$\frac{x^2 - 2x + 1}{x^2 - x}.$$

5. Find the solution(s) of $x^2 - x = 12$. Give exact answer(s).

6. Find the solution(s) of $x^2 + x - 1 = 0$. Give exact answer(s).

7. Solve $\frac{1}{2}(x+2) + \frac{1}{3}(x+6) = 2$.

8. Find the equation of the line that passes through $(x, y) = (2, 4)$ and $(x, y) = (-1, 9)$. Simplify to put the equation in the form $y = mx + b$.

9. If Lola runs 8 kilometers per hour for 20 minutes, how far will she run?

10. Write in the form $x^a y^b$.

$$xy^2(x^3y)^2$$

Solutions to pretest for Calculus I

Give one point per problem. Generally, problems should be marked right or wrong. You may award 1/2 point for students who solve a problem correctly but make a minor mistake in arithmetic. Please do not duplicate these answers and distribute to students. We reuse this pretest. It would not be helpful for students to artificially improve their scores by studying the solutions in future semesters.

1. Write as a single rational expression by obtaining a common denominator.

$$\frac{1}{x+1} - \frac{1}{x-1}.$$

Answer: $\frac{-2}{(x-1)(x+1)}$.

2. If the two shorter sides of a right triangle are of length 2 and 3, what is the length of the hypotenuse?

Answer: $\sqrt{13}$.

3. Simplify by cancelling common factors.

$$\frac{\cos x \sin x + \sin^2 x}{\sin^2 x}.$$

Answer: $\frac{\cos(x)+\sin(x)}{\sin(x)}$.

4. Factor the numerator and denominator and cancel common factors.

$$\frac{x^2 - 2x + 1}{x^2 - x}.$$

Answer: $\frac{x-1}{x}$

5. Find the solution(s) of $x^2 - x = 12$. Give exact answer(s).

Answer: $x = 4$ or $x = -3$.

6. Find the solution(s) of $x^2 + x - 1 = 0$. Give exact answer(s).

Answer: $x = \frac{-1 \pm \sqrt{5}}{2}$

7. Solve $\frac{1}{2}(x+2) + \frac{1}{3}(x+6) = 2$.

Answer: $-6/5$.

8. Find the equation of the line that passes through $(x, y) = (2, 4)$ and $(x, y) = (-1, 9)$. Simplify to put the equation in the form $y = mx + b$.

Answer: $y = \frac{-5}{3}x + \frac{22}{3}$.

9. If Lola runs 8 kilometers per hour for 20 minutes, how far will she run?

   Answer: 8/3 kilometers.

10. Write in the form $x^a y^b$.

$$xy^2(x^3y)^2$$

    Answer: x^7y^4

To: Ma113 instructors.
From: Russell Brown
Date: 8 January 2007

Subject: Pretest guidelines

All students will take a pretest in MA 113 this spring. Below are guidelines to help students interpret their scores.

I recommend that students who score 8 or above on the pretest be encouraged to continue in Calculus. Students who score below 5 should be discouraged from continuing in MA 113. Students who score 5 to 7.5 should be warned that there may be deficiencies in their background. If they continue in MA 113 they should plan on doing extra work to succeed in this course. This extra work might include 1) working the warm-up assignments (AW, BW, CW and DW), 2) routine exercises from the book, 3) joining the MathExcel section. Students interested in the MathExcel section should contact Brown or Shen. These recommendations should come from the lecturer, rather than a teaching assistant.

The table below summarizes the performance in MA113, Calculus I, of a number of students who have taken the pretest. Success is defined to mean earning A, B or C. The information comes from 366 students in 19 sections over 4 semesters and 5 instructors. Note that this table only includes students who finish the course with ABCDEW. By looking at students who choose to stay in the course, it is likely that the probability of success is artificially high for students who obtain a low score on the pretest.

Score	Prob of succ	Fraction of class
10	89%	10%
9-9.5	80%	17%
8-8.5	77%	16%
7-7.5	80%	17%
6-6.5	49%	11%
5-5.5	50%	13%
4-4.5	32%	8%
3-3.5	55%	5%
0-2.5	7%	4%