Calculus I *Spring 2006*
MA113

Textbook: The textbook for this course will be *Calculus*, 5th edition, by James Stewart.

Material to be covered: In Calculus I, we will learn about derivatives, integrals and the fundamental theorems of calculus. We begin by introducing the notion of a limit. Limits are essential to defining derivatives and integrals. By the end of the semester students should know precise definitions of the derivative and the integral and the fundamental theorem of calculus which gives the relation between the derivative and the integral. We will illustrate the methods and ideas of calculus by studying several physical and geometric problems. We will study the interpretation of the derivative as velocity or slope of a tangent line, the trajectory of a body falling under the influence of gravity, the interpretation of the integral as area or distance traveled and the use of the integral in computing volumes of familiar solids such as a sphere or a cone.
We will cover most of Chapters 1 to 6 of Stewart. Please see the course calendar for a detailed listing of sections.

Homework: The bulk of homework for this course will be completed using the web-based homework system at `http://www.mathclass.org`. Most students should already have an account at this site. Your user id and password are both equal to your student id number. Students who registered late or changed sections will not have an account and should go to this site, create an account and request registration in the class titled `MA113-nnn` where `nnn` is your section number. Be sure that you are a registered student and not just browsing. Information on using this web-site is available by clicking the link titled "For students" and then following the link for the tutorial or by following the Help link which appears on many WHS pages.
Students must drop MA 113 through Web UK. Dropping your registration in the web-based homework system will have no effect on your official registration. Students who change sections will need to speak with their lecturer. Students cannot change sections in the web-based homework system. The administrator of the system can transfer students, if necessary.
Homework will be discussed in recitation on Tuesday and Thursday and submitted by 12 midnight on the following Monday. Students should attempt homework as soon as the corresponding material is discussed in lecture. Students who wait till Monday to begin an assignment will likely not complete the work on time.
Each student will have an individual version of the homework. Students should plan to print out their assignment, complete the problems on a separate piece of paper or in a notebook, submit their answers and then rework or seek assistance on the problems that were marked incorrect. Your teaching assistants will be instructed to ask to see your work before providing assistance. In addition, there is a common version of each homework set. The problems from the common version will be discussed in recitation.
If you feel you have worked a problem correctly and WHS marks it incorrect, please let contact Russell Brown (by e-mail to `russell.brown@uky.edu` or by submitting the form

at http://www.math.uky.edu/~rbrown/whs/report.html).

In addition to the web homework, we will have seven worksheets that will be graded by humans. These worksheets will be graded for mathematical correctness and for clarity of exposition. Students who wish to receive full credit should write in complete sentences and use mathematical notation correctly.

The course calendar lists optional homework assignments from the textbook. These are intended for students who feel they need more practice to master a topic.

The homework grade in the course is computed as follows. The web homework grade is the minimum of 95 and your average score on web homework. You may find this average in WHS by selecting Homework Scores on the main page. Add the web homework grade and the grades on the seven worksheets to obtain the total homework points earned. The homework grade is the percentage of points that are earned out of the 165 possible points.

Late homework: No late submissions of web homework will be accepted. If an emergency or illness takes you away from school, please meet with your lecturer to discuss your situation and ask to be excused from an assignment, if appropriate. If you have a scheduled absence (travel or authorized university absence) you must still submit the homework by the deadline.

Written assignments are due at the beginning of lecture. If an emergency or unexpected absence prevents you from turning in the assignment, please see your lecturer to request permission to turn in the assignment late. If you have a scheduled absence (travel or authorized university absence) you should arrange to turn in your paper before leaving school. Unexcused and late submissions will be penalized 10% if the paper is turned in on the due date and an additional 20% for each day that it is late.

Exams: There will be three exams and a final. These exams are scheduled in the evening as indicated in the course calendar. Please be sure that you have these dates free. The final exam will be cumulative, but with an emphasis on the material covered since the last test.

MA193: In addition, to the 4 hours of credit for MA113, the department offers one additional hour of credit as MA193 on a pass/fail basis. You will pass MA193 if you have 0, 1 or 2 unexcused absences and you pass MA113. If you have three or more unexcused absences or you fail MA 113, you will fail MA193. Your section number for MA193 should equal your section number for MA113. If you drop or change sections of MA113, please make sure to also drop or change sections of MA193.

Grading: Your grade will be based on the activities in the table below.

3 exams	300
Final exam	100
Homework	100
TOTAL	500

Students need an average of 90% (450 points) for an A, 80% (400 points) for a B, 70% (350 points) for a C and 60% (300 points) for a D. Grades may be curved by lowering these grade lines.

Calculators: Students may use a graphing calculator on exams and homework. Students may not use a machine with symbolic manipulation capabilities on exams. Thus, no TI-89's, TI-92's, no HP-48's or laptop computers may be used on exams. Please see the lecturer if you have any questions as to whether a particular machine may be used on a test.

Absences: You should attend class. If you must miss a recitation and are registered for MA193, you must explain your absence to your teaching assistant. Otherwise, your absence will be marked as unexcused and this may lead to failing MA193.

Web page: A web page for this course is at `http://www.math.uky.edu/~rbrown/courses/ma113.s.06` Any handouts will be available at this address.

Assistance: Teaching assistants and tutors are available in Mathskeller to help with Calculus. This resource center is located in the basement of the Classroom Building. Mathskeller will be open Monday–Friday 10–5. In addition to tutoring, Mathskeller is a convenient place to print out homework assignments and to obtain assistance in registering with the online homework system. Tutoring organized by Mathskeller is available Sunday–Thursday, 6–10 pm in room B-25 of Young Library Visit `http://www.mathskeller.com` to find changes to this schedule around holidays and common exams.

Date	Section, topic, assignments.	Assignments (textbook problems are optional)
11-Jan	Ch. 1 Review of functions	Review, p. 56 #1,2,3,5,6,8-12,16-19
12-Jan	Pretest, Assignment A1.	A1: Review
13-Jan	Mathematical induction, handout.	
16-Jan	Martin Luther King, Jr. Day	
17-Jan	Worksheet 1.	
18-Jan	2.1 Tangent and velocity problems	§2.1 #1,2,3,5,6,8,9
	2.2 Limit of a function	§2.2, #2,4,5,6,9,12,13,15,25,28
19-Jan	Assignment A2, practice quiz 1	A2: Tangents and velocity
20-Jan	2.3 Calculating limits using the limit laws	§2.3 #1,2,11,13,15,17,20,22,28,39,40,49
	2.4 The precise definition of a limit (lightly)	§2.4 #1,2,3,4,5,6,9,10
23-Jan	2.5 Continuity	§2.5, #1,3,4,5,6,7,9,11,17,21,37
	12m submission deadline for A1 and A2	
24-Jan	Assignment A3, Practice quiz 2	A3: Limits
25-Jan	2.6 Tangents, velocities and rates of change	§2.6 #1,2,3,5,13,15,17,18,23
	Worksheet 1 due in class	
26-Jan	Assignment A4, Worksheet 2.	A4: Tangents, velocity, rates of change
27-Jan	3.1 Derivatives	§3.1 #3,4,6,7,9,12,15,16,19,22,25,26
30-Jan	3.2 The derivative as a function	§3.2 #1,2,4,7,10,12,17,25,36,39,41
	12m submission deadline for A3 and A4.	
31-Jan	Assignment A5	A5: The derivative
1-Feb	3.3 Differentiation formula	§3.3 #5,10,16,18,21,25,28,33,40,44,53,57,58,62
	Worksheet 2 due in class	
	Last day to drop	
2-Feb	Assignment A6, practice quiz 3	A6: Differentiation rules
3-Feb	Review	
6-Feb	Review	
	12m submission deadline for A5 and A6.	
7-Feb	R1 (not graded)	
	First exam, 7:30-9:30pm room TBA.	
8-Feb	Appendix D, Trigonometry	Appendix D, #1,4,7,10,13,15,23,26,29,30,31,
9-Feb	Assignment B1	B1: Trigonometry review
10-Feb	3.5 Derivatives of trigonometric functions	§3.5 #3,6,9,12,18,29,30,35,36,43
13-Feb	3.6 The chain rule	§3.6 #1,5,6,7,10,15,16,19,25,28,45,46,55,56
	12m submission deadline for B1.	
14-Feb	Assignment B2, worksheet 3	B2: Derivatives of trigonometric functions
15-Feb	3.7 Implicit differentiation	§3.7 #3,4,7,10,14,15,26,29,35,39
16-Feb	Assignment B3, practice quiz 4	B3: The chain rule
17-Feb	3.8 Higher derivatives	§3.8 #1-3,11,18,25,26,39,41,44,49,50,53
20-Feb	3.9 Related rates	§3.9 #1,2,4,6-8,10-12,14-17,20-22
	12m submission deadline for B2 and B3.	
21-Feb	Assignment B4, practice quiz 5	B4: Implicit differentiation and higher order derivat
22-Feb	3.9 continued	
	Worksheet 3 due in class.	
23-Feb	Assignment B5, worksheet 4	B5: Related rates
24-Feb	3.10 Linear approximation	§3.10 #1,3,7,8,13,15,27,31,32,37
27-Feb	4.1 Maximum and minimum values	§4.1 #1,2,3,4,5,9,11,17,18,23,47,48,52
	12m submission deadline for B4 and B5.	
28-Feb	Assignment B6.	B6: Linear approximation
1-Mar	4.2 The mean value theorem	§4.2 #1,3,5-8,15-19,22
	Worksheet 4 due in class.	
2-Mar	Assignment B7, practice quiz 6	B7: Extreme values and the mean value theorem
3-Mar	Review	
6-Mar	Review	
	12m submission deadline for B6 and B7.	
7-Mar	R2 (not graded)	
	7:30-9:30 pm Exam 2, room TBA	

8-Mar	4.3 Derivatives and the shape of a graph	§4.3 #1,2,5,6,7-9,11-17,22-26,29,31,33
7-Mar	Assignment C1	C1: Derivatives and the shape of a graph
10-Mar	4.4 Limits at infinity	§4.4 #1-4,9,11,13,15,17,19,21,23,35,37,39,43,58
	Last day to withdraw	
13-18 Mar	Spring break	
20-Mar	4.5 Summary of curve sketching	§4.5 #3,12,13,17,23,27,31
	12m submission deadline for C1.	
21-Mar	Assignment C2, worksheet 5	C2: Summary of curve sketching
22-Mar	4.5 continued	§4.6 #20,21,26,27
23-Mar	Practice quiz 7	
24-Mar	4.7 Optimization problems	§4.7 #2,3,6,7,10,16,19,22,29,32,35,51,52.
27-Mar	4.7 continued	
	12m submission deadline for C2	
28-Mar	Assignment C3, practice quiz 8	C3: Optimization
29-Mar	4.9 Newton's method	§4.9 #1,4,5,6,11,27,31,34,35
	Worksheet 5 due in class.	
30-Mar	Assignment C4. worksheet 6.	C4: Newton's method
31-Mar	4.10 Anti-derivatives	§4.10 #1,3,5,7,21,23,25,31,36,37,39,40,53,55,68,
3-Apr	5.1 Areas and distances	§5.1 #1,3,4,5,11,12,20,22
	12m submission deadline for C3 and C4.	
4-Apr	Assignment C5.	C5: Anti-derivatives
5-Apr	5.2 The definite integral	§5.2 #1,7,9,17,19,25,29,30,33-36,39,47-49,55,57
	Worksheet 6 due in class.	
6-Apr	Assignment C6, practice quiz 9	C6: Areas and distances: the definite integral
7-Apr	Review	
10-Apr	Review	
	12m submission deadline for C5 and C6	
11-Apr	R3 (not graded)	
	7:30pm-9:30pm, room TBA.	
12-Apr	5.3 The fundamental theorem of calculus	§5.3 #1,7-11,13,19,21,23,25,27,31,33,51,
13-Apr	Assignment D1	D1: The fundamental theorem of calculus
14-Apr	5.4 Indefinite integrals	§5.4 #1,3,17,19,21,23,25,31,33,43,46,48
17-Apr	5.5 Substitution	§5.5 #1,3,9,11,13,15,17,19,21,27,37,39,45,49,57,
	12m submission deadline for D1	
18-Apr	Assignment D2, practice quiz 10	D2: Substitution
19-Apr	6.1 Areas between curves	§6.1 #1,2,5,7,11,13,21,22,24,45
20-Apr	Assignment D3, worksheet 7	D3: Area
21-Apr	6.2 Volume	§6.2 #1,3,12,13,14,47,48,49,53
24-Apr	6.3 Volume by cylindrical shells	§6.3 #1,9,11,13,15,17,43,45.
	12m Submission deadline for D2 and D3	
25-Apr	Assignment D4	D4: Volumes
26-Apr	Review	
	Worksheet 7 due in class	
27-Apr	Assignment R4 (not graded)	
	12m submission deadline for D4	
28-Apr	Review	
1-May	Final exam, 6-8pm room TBA	

Worksheet on induction
Calculus I

MA113
Spring 2006

First, let us explain the use of the $\sum$ for summation. The notation

$$\sum_{k=1}^{n} f(k)$$

means to evaluate the function $f(k)$ at $k = 1, 2, \ldots, n$ and add up the results. In other words:

$$\sum_{k=1}^{n} f(k) = f(1) + f(2) + \ldots + f(n).$$

For example:

$$\sum_{k=1}^{4} k^2 = 1 + 4 + 9 + 16,$$

$$\sum_{k=1}^{n} (2k-1) = 1 + 3 + 5 + \ldots + 2n - 1,$$

and

$$\sum_{k=3}^{2n} 1 = 2n - 2.$$

The principle of mathematical induction is used to establish the truth of a sequence of statements or formula which depend on a natural number, $n = 1, 2, \ldots$. We will use P_k to stand for a statement which depends on k. For example, P_k might stand for the statement "The number $2k - 1$ is odd." This sequence of statements is true for $k = 1, 2, \ldots$

The principle of mathematical induction is:

Principle of mathematical induction. Suppose that P_n is a sequence of statements depending on a natural number $n = 1, 2, \ldots$. If we show that:

- P_1 is true
- For $N = 1, 2, \ldots$: If P_N is true, then P_{N+1} is true.

Then, we may conclude that all the statements P_n are true for $n = 1, 2, \ldots$.

To see why this holds, suppose that we know P_1 is true, then the second step allows us to conclude P_2 is true. Now that we know P_2 is true, the second step allows us to conclude P_3 is true. If we repeat this $n - 1$ times, we know that P_n is true.

This principle is useful because it allows us to prove an infinite number of statements are true in just two easy steps! We usually call the first step the *base case* and the second step is called the *induction step.*

Below are several examples to illustrate how to use this principle. The statement P_N that we assume to hold is called the *induction hypothesis.* The key point in the induction step is to show how the truth of the induction hypothesis, P_N, leads to the truth of P_{N+1}.

Example 1. Show that for $n = 1, 2, 3, \ldots$, the number $4^n - 1$ is a multiple of 3.
Solution. Base case. We need to show this is true when $n = 1$. This is easy since $4^1 - 1 = 4 - 1 = 3$ and 3 is divisible by 3.
Induction step. We suppose that $4^N - 1$ is a multiple of 3 and we want to use this assumption to show that $4^{N+1} - 1$ is a multiple of 3. Our assumption for N means that for some whole number M, $4^N - 1 = 3M$. Now $4^{N+1} - 1$. If we add and subtract 4, we have

$$4^{N+1} - 1 = 4^{N+1} - 4 + 4 - 1 = 4(4^N - 1) + 3.$$

Now we use our induction hypothesis that $4^N - 1$ is a multiple of 3 to replace $4^N - 1$ by $3M$ and obtain that

$$4^{N+1} - 1 = 4 \cdot 3M + 3 = 3(4M + 1).$$

Thus we have shown that $4^{N+1} - 1$ is a multiple of 3.

Example 2. Show that for $n = 1, 2, \ldots$, we have

$$\sum_{j=1}^{n} 2j = n(n+1).$$

Solution Base case. If $n = 1$, then $n(n+1) = 1 \cdot 2 = 2$. Also,

$$\sum_{j=1}^{1} 2j = 2.$$

Thus both sides are equal if $n = 1$.
Induction step. Now suppose that the formula is true for N and consider the sum

$$\sum_{j=1}^{N+1} 2j = \sum_{j=1}^{N} 2j + 2(N+1).$$

We use our induction hypothesis that $\sum_{j=1}^{N} 2j = N(N+1)$ to conclude that

$$\sum_{j=1}^{N+1} 2j = N(N+1) + 2(N+2).$$

Simplifying this last expression gives

$$N(N+1) + 2(N+1) = N^2 + N + 2N + 2 = N^2 + 3N + 2 = (N+2)(N+1).$$

Since $(N+2)(N+1) = (N+1+1)(N+1)$, we have shown that the formula

$$\sum_{j=1}^{N+1} 2j = (N+1+1)(N+1)$$

is true. This completes the proof by induction.

Below is a selection of problems related to mathematical induction. You should begin working on these problems in recitation. Write up your solutions carefully, elegantly, and in complete sentences.

1. (a) For $n = 1, 2, 3, 4$, compute

$$\sum_{k=1}^{n} (2k-1).$$

   Make a guess for the value of this sum for $n = 1, 2, \ldots$.

   (b) Use mathematical induction to prove that your guess is correct.

2. Use mathematical induction to prove that

$$\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6.$$

3. Let $f_1(x) = x - 2$ and then define f_n for $n = 1, 2, \ldots$ by $f_{n+1}(x) = f_1(f_n(x))$. (It is the principle of mathematical induction which tells us that these two statements suffice to define f_n for all n.) Use mathematical induction to prove that

$$f_n(x) = x - 2n.$$

4. Let P_n be the statement: $n^2 - n$ is an odd integer.

   (a) Show that if P_n is true, then P_{n+1} is true.

   (b) Is P_1 true?

   (c) Is P_n true for any n?

Below are some additional exercises. You may not be able to solve all of these problems at this time. These problems will not be collected.

1. Let $f(x) = \sin(2x)$. Prove that for $n = 1, 2, \ldots$,

$$\frac{d^{2n}}{dx^{2n}} f(x) = (-4)^{2n} \sin(2x).$$

2. Prove that

$$\frac{d}{dx}x^n = nx^{n-1}, \qquad n = 1, 2 \ldots.$$

Hint: For the base case $n = 1$, use the definition of the derivative. For the induction step write $x^{n+1} = x \cdot x^n$ and use the product rule.

3. Prove that

$$\frac{d}{dx}\frac{1}{x^n} = \frac{-n}{x^{n+1}}, \qquad n = 1, 2 \ldots.$$

4. Prove that

$$\frac{d^n}{dx^n}x^n = n!, \qquad n = 0, 1, \ldots$$

5. (a) Find a simple formula for

$$\sum_{k=1}^{n}((k+1)^2 - k^2) = 2^2 - 1 + (3^2 - 2^2) + \ldots + n^2 - (n-1)^2 + (n+1)^2 - n^2.$$

(b) Using your answer to part a), find a simple expression for

$$\sum_{k=1}^{n}(2k-1).$$

To do this you should simplify each summand on the left.

6. Use mathematical induction to prove that

$$\sum_{j=1}^{n} j^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

January 6, 2006

Before beginning, it might be helpful to recall the quadratic formula. The roots of the quadratic equation $ax^2 + bx + c = 0$ are

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The quantity inside the radical, $b^2 - 4ac$, is called the *discriminant*. It is easy to see that we have two real roots if the discriminant is positive, one real root if the discriminant is 0 and no real real roots if the discriminant is zero.

1. Find the slope of the tangent line to the graph of $f(x) = x^2$ at a general point $x = a$. Use the definition.

2. If we consider the parabola $y = x^2$, and a point (c, d) in the plane, how many tangent lines to the parabola are there that pass through (c, d) (which may not lie on the parabola)? The exercises below answer this question and allow you to relate the number of tangent lines to the location of the point.

   (a) Make three sketches which show the tangent line(s) to $y = x^2$ which pass through

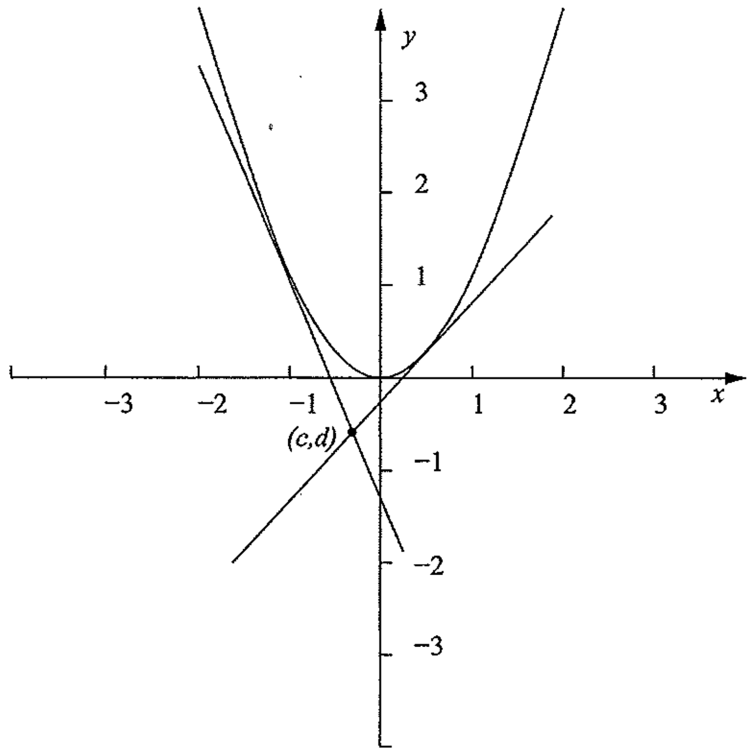
      i. $(1, -2)$
      ii. $(1, 1)$
      iii. $(0, 1)$.

   (b) Make a conjecture as to how many tangent lines of the parabola pass through a given point (c, d). How does the answer depend on the point (c, d)?

   (c) Write the equation of the tangent line to the parabola $y = x^2$ at (a, a^2).

   (d) If we require the tangent line in part c) to pass through point (c, d), we obtain an equation for a the x-coordinate of the point where the tangent line meets the parabola. Write out this equation.

   (e) Tell how many solutions the equation you found in part d) has. How does the number of solutions depend on (c, d)? The best answers to this question will refer to the discriminant of a quadratic equation. Can you interpret your answer geometrically?



This problem was suggested by Jim Brennan.

January 5, 2006