Attached is a revised syllabus packet. After I prepared the packet for instructors, I found and corrected a few errors in the course calendar and edited the instructions to the student for accessing ~~mathskeller~~ mathdaxx.org.

Syllabus packet

1. Syllabus
2. Calendar
3. Suggestions
4. Common errors
5. Worksheets 1 and 2.

Instructor information

1. Outline of TA and lecturer duties.
2. A suggestion for the first recitation.
3. Pretest, solutions and summary statistics.

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Calculus I *Fall 2006*
MA113

Textbook: The textbook for this course will be *Calculus*, 5th edition, by James Stewart.

Material to be covered: In Calculus I, we will learn about derivatives, integrals and the fundamental theorems of calculus. We begin by introducing the notion of a limit. Limits are essential to defining derivatives and integrals. By the end of the semester students should know precise definitions of the derivative and the integral and the fundamental theorem of calculus which gives the relation between the derivative and the integral. We will illustrate the methods and ideas of calculus by studying several physical and geometric problems. We will study the interpretation of the derivative as velocity or slope of a tangent line, the trajectory of a body falling under the influence of gravity, the interpretation of the integral as area or distance traveled and the use of the integral in computing volumes of familiar solids such as a sphere or a cone.
We will cover most of Chapters 1 to 6 of Stewart. Please see the course calendar for a detailed listing of sections.

Homework: The bulk of homework for this course will be completed using the web-based homework system at `http://www.mathclass.org`. Students who pre-registered over the summer will have an account at this website. Your user name for this account is the e-mail address that you have on file with the University. The password will be the last 6 digits of your social security number followed by an exclamation point, *e.g.* 654321!. If you have difficulty logging in, you may visit Mathskeller (CB 065) M–F from 9am–4pm. In addition, your instructor or another student with an account at `mathclass.org` may look up the e-mail address using the builtin directory.
Students who registered near the beginning of the semester may not have an account. These students will need to go to `www.mathclass.org`, and create an account. After logging on to `www.mathclass.org`, click on the triangle next to `Tools`, then `Web homework` and request to add the class titled `MA113-nnn` where `nnn` is your section number. Be sure that you are a registered student and not just browsing. Information on using `mathclass.org` is available in the guides which are available from links on the left-hand side of `www.mathclass.org`.
Students who choose to drop MA 113 must use Web UK. Dropping your registration at `www.mathclass.org` will have no effect on your official registration. Students who switch sections of MA 113 during add-drop should also add their new section at `mathclass.org`. Students who are switching sections should not create a new account. When a student adds a section of MA 113 to their account at `mathclass.org`, the record of homework will be transferred to the new section.
Web homework problems will be discussed in recitation on Tuesday and Thursday and submitted by 12 midnight on the following Monday. Students should attempt homework as soon as the corresponding material is discussed in lecture. Students who wait till Monday to begin an assignment will likely not complete the work on time.
Each student will have an individual version of the homework. Students should plan to

print out their assignment, complete the problems in a notebook, submit their answers and then rework or seek assistance on the problems that were marked incorrect. Your teaching assistants will be instructed to ask to see your work before providing assistance. In addition, there is a common version of each homework set. The problems from the common version will be discussed in recitation.

If you feel you have worked a problem correctly and WHS marks it incorrect, please contact Russell Brown (by e-mail to `russell.brown@uky.edu` or by submitting the form at `http://www.math.uky.edu/~rbrown/whs/report.html`).

There are several web homework assignments that will not be counted towards your grade. The review assignments R1-R4 are study guides for each exam. All students should complete these review assignments. The warmup assignments W1-W4 are collections of routine problems that are provided for students who would like additional practice on basic skills.

The course calendar lists optional homework assignments from the textbook. These are intended for students who feel they need more practice to master a topic.

In addition to the web homework, we will have seven worksheets that will be graded by humans. These worksheets will be graded for mathematical correctness, and for clarity of exposition. Students who wish to receive full credit should write in complete sentences and use mathematical notation correctly.

The homework grade in the course is computed as follows. The web homework grade is the minimum of 95 and your average score on web homework. You may find this average at `mathclass.org` by clicking homework scores after logging in. Add the web homework grade and the grades on the seven worksheets to obtain the total homework points earned. The homework grade is the percentage of points that are earned out of the 165 possible points.

Late homework: No late submissions of web homework will be accepted. If an emergency or illness takes you away from school, please meet with your lecturer to discuss your situation and ask to be excused from an assignment, if appropriate. If you have a scheduled absence (travel or authorized university absence) you must still submit the homework by the deadline.

Written assignments are due at the beginning of lecture. If an emergency or unexpected absence prevents you from turning in the assignment, please see your lecturer to request permission to turn in the assignment late. If you have a scheduled absence (travel or authorized university absence) you should arrange to turn in your paper before leaving school. Unexcused and late submissions will be penalized 10% if the paper is turned in on the due date and an additional 20% for each day that it is late.

Exams: There will be three exams and a final. These exams are scheduled in the evening as indicated in the course calendar. Please be sure that you have these dates free. The final exam will be cumulative, but with an emphasis on the material covered since the third exam.

MA193: In addition, to the 4 hours of credit for MA113, the department offers one additional hour of credit as MA193 on a pass/fail basis. You will pass MA193 if you have

0, 1 or 2 unexcused absences and you pass MA113. If you have three or more unexcused absences or you fail MA 113, you will fail MA193. Your section number for MA193 should equal your section number for MA113. If you drop or change sections of MA113, please make sure to also drop or change sections of MA193.

Grading: Your grade will be based on the activities in the table below.

3 exams	300
Final exam	100
Homework	100
TOTAL	500

Students need an average of 90% (450 points) for an A, 80% (400 points) for a B, 70% (350 points) for a C and 60% (300 points) for a D. Grades may be curved by lowering these grade lines.

Calculators: Students may use a graphing calculator on exams and homework. Students may not use a machine with symbolic manipulation capabilities on exams. Thus, no TI-89's, TI-92's, no HP-48's or laptop computers may be used on exams. Please see the lecturer if you have any questions as to whether a particular machine may be used on a test.

Absences: You should attend class. If you must miss a recitation and are registered for MA193, you must explain your absence to your teaching assistant. Otherwise, your absence will be marked as unexcused and this may lead to failing MA193.

Web page: A web page for this course is at
`http://www.math.uky.edu/~rbrown/courses/ma113.f.06` Any handouts will be available at this address.

Date	Section, topic, assignments.	Assignments (textbook problems are optional)
23-Aug	Ch. 1 Review of functions	Review, p. 56 #1,2,3,5,6,8-12,16-19
24-Aug	Pretest, Assignment A1.	A1: Review
25-Aug	Mathematical induction, handout.	
28-Aug	2.1 Tangent and velocity problems	§2.1 #1,2,3,5,6,8,9
29-Aug	Worksheet 1.	
30-Aug	2.2 Limit of a function	§2.2, #2,4,5,6,9,12,13,15,25,28
31-Aug	Assignment A2, practice quiz 1	A2: Tangents and velocity
1-Sep	2.3 Calculating limits using the limit laws	§2.3 #1,2,11,13,15,17,20,22,28,39,40,49
4-Sep	Labor day, academic holiday	
5-Sep	Assignment A3	A3: Limits
	12m submission deadline for A1 and A2	
6-Sep	2.5 Continuity	§2.5, #1,3,4,5,6,7,9,11,17,21,37
	Worksheet 1 due in class.	
7-Sep	Assignment A4, practice quiz 2	A4: Continuity
8-Sep	2.6 Tangents, velocities and rates of change	§2.6 #1,2,3,5,13,15,17,18,23
11-Sep	3.1 Derivatives	§3.1 #3,4,6,7,9,12,15,16,19,22,25,26
	12m submission deadline for A3 and A4.	
12-Sep	Assignment A5, worksheet 2	A5: Tangents velocity, rates of change
13-Sep	3.2 The derivative as a function	§3.2 #1,2,4,7,10,12,17,25,36,39,41
	Worksheet 2 due in class	
	Last day to drop	
14-Sep	Assignment A6, practice quiz 3	A6: The derivative
15-Sep	Review	
18-Sep	Review	
	12m submission deadline for A5 and A6.	
19-Sep	R1 (not graded)	
	First exam, 7:30-9:30pm room TBA.	
20-Sep	Appendix D, Trigonometry	Appendix D, #1,4,7,10,13,15,23,26,29,30,31,
21-Sep	Assignment B1	B1: Review of trigonometry
22-Sep	3.3 Differentiation formula	§3.3 #5,10,16,18,21,25,28,33,40,44,53,57,58,62
25-Sep	3.5 Derivatives of trigonometric functions	§3.5 #3,6,9,12,18,29,30,35,36,43
	12m submission deadline for B1.	
26-Sep	Assignment B2, worksheet 3	B2: Differentiation rules
27-Sep	3.6 The chain rule	§3.6 #1,5,6,7,10,15,16,19,25,28,45,46,55,56
28-Sep	Assignment B3, practice quiz 4	B3: Differentiation of trigonometric functions
29-Sep	3.7 Implicit differentiation	§3.7 #3,4,7,10,14,15,26,29,35,39
2-Oct	3.8 Higher derivatives	§3.8 #1-3,11,18,25,26,39,41,44,49,50,53
	12m submission deadline for B2 and B3.	
3-Oct	Assignment B4, practice quiz 5	B4: The chain rule
4-Oct	3.9 Related rates	§3.9 #1,2,4,6-8,10-12,14-17,20-22
	Worksheet 3 due in class.	
5-Oct	Assignment B5, worksheet 4	B5: Implicit differentiation, higher order derivatives
6-Oct	Fall break, academic holiday	
9-Oct	3.10 Linear approximation	§3.10 #1;3,7,8,13,15,27,31,32,37
	12m submission deadline for B4 and B5.	
10-Oct	Assignment B6.	B6: Related rates
11-Oct	4.1 Maximum and minimum values	§4.1 #1,2,3,4,5,9,11,17,18,23,47,48,52
	Worksheet 4 due in class.	
12-Oct	Assignment B7, practice quiz 6	B7: Linear approximation, Extreme values
13-Oct	Review	
16-Oct	Review	
	12m submission deadline for B6 and B7.	
17-Oct	R2 (not graded)	
	7:30-9:30 pm Exam 2, room TBA	

Date	Section, topic, assignments.	Assignments (textbook problems are optional)
18-Oct	4.2 The mean value theorem	§4.2 #1,3,5-8,15-19,22
19-Oct	Assignment C1	C1: The mean value theorem
20-Oct	4.3 Derivatives and the shape of a graph	§4.3 #1,2,5,6,7-9,11-17,22-26,29,31,33
	Last day to withdraw	
23-Oct	4.4 Limits at infinity	§4.4 #1-4,9,11,13,15,17,19,21,23,35,37,39,43,58
	12m submission deadline for C1.	
24-Oct	Assignment C2, worksheet 5	C2: Derivatives and the shape of a graph
25-Oct	4.5 Summary of curve sketching	§4.5 #3,12,13,17,23,27,31
26-Oct	Practice quiz 7	C3: Summary of curve sketching
27-Oct	4.5 continued	§4.6 #20,21,26,27
30-Oct	4.7 Optimization problems	§4.7 #2,3,6,7,10,16,19,22,29,32,35,51,52.
	12m submission deadline for C2 and C3.	
31-Oct	Assignment C3, practice quiz 8	C4: Optimization
1-Nov	4.9 Newton's method	§4.9 #1,4,5,6,11,27,31,34,35
	Worksheet 5 due in class.	
2-Nov	Assignment C4. worksheet 6.	C5 Newton's method
3-Nov	4.10 Anti-derivatives	§4.10 #1,3,5,7,21,23,25,31,36,37,39,40,53,55,68,7
6-Nov	5.1 Areas and distances	§5.1 #1,3,4,5,11,12,20,22
	12m submission deadline for C4 and C5.	
7-Nov	Assignment C6..	C6: Anti-derivatives
8-Nov	5.2 The definite integral	§5.2 #1,7,9,17,19,25,29,30,33-36,39,47-49,55,57
	Worksheet 6 due in class.	
9-Nov	Assignment C7, practice quiz 9	C7: Areas and distances: the definite integral
10-Nov	Review	
13-Nov	Review	
	12m submission deadline for C6 and 7.6	
14-Nov	R3 (not graded)	
	7:30pm-9:30pm, room TBA.	
	Gottfried Wilhelm Leibniz died, 1716	
15-Nov	5.3 The fundamental theorem of calculus	§5.3 #1,7-11,13,19,21,23,25,27,31,33,51,
16-Nov	Assignment D1	D1: The fundamental theorem of calculus
17-Nov	5.4 Indefinite integrals	§5.4 #1,3,17,19,21,23,25,31,33,43,46,48
20-Nov	5.5 Substitution	§5.5 #1,3,9,11,13,15,17,19,21,27,37,39,45,49,57,5
	12m submission deadline for D1	
21-Nov	Assignment D2, practice quiz 10	D2: Indefinite integrals, substitution
22-Nov	5.5 Substitution, continued	
23-Nov	Thanksgiving break, academic holiday	
24-Nov	Thanksgiving break, academic holiday	
27-Nov	6.1 Areas between curves	§6.1 #1,2,5,7,11,13,21,22,24,45
28-Nov	Assignment D3, worksheet 7	D3: Area
	12m submission deadline D2	
29-Nov	6.2 Volume	§6.2 #1,3,12,13,14,47,48,49,53
30-Nov	Assignment D4, practice quiz 11	D4: Volumes
1-Dec	6.3 Volume by cylindrical shells	§6.3 #1,9,11,13,15,17,43,45.
4-Dec	Review	
	12m submission deadline for D3	
5-Dec	Assignment D4, continued	
6-Dec	Review	
	Worksheet 7 due in class	
7-Dec	Assignment R4 (not graded)	
	12m submission deadline for D4	
8-Dec	Review	
11-Dec	Final exam, 6-8pm room TBA	

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Some suggestions on how to study mathematics.

- Go to class.
- Take notes. The notes you produce will provide a written record of what your instructor believes to be the most important points of the course. Your notes will provide additional examples that will be helpful as you solve problems on homework. Writing notes will help focus your attention on the mathematics being presented in class.
- Go to class when exams and written assignments are returned. Be sure to collect every assignment. Review mistakes in your work so that you will be less likely to repeat the mistake on the next assignment.
- Read the textbook before you go to class. While you may not understand every detail after reading a section in the text, you will be familiar with the main ideas and class will help to deepen your understanding.
- Write careful and complete solutions to each homework problem and keep these solutions in a notebook. We will not, in general, grade your solutions. Your solutions will be useful when you review for exams. Your solutions will be helpful when you seek assistance on a problem and need to explain how you have approached the problem.

  In your work, your goal is not just to arrive at a correct answer but to also to explain your reasoning in order to convince the reader that you understand how to solve the problem.
- Form a study group. After you go to class, work on homework with fellow students. Make sure that you are able to answer every question. After you have answered every question, make sure that you understand why your answers are correct. Discuss your reasoning with your fellow students and see if others have a better approach to solving a problem.
- Attempt homework problems immediately after material is covered in class. Mark problems that you find difficult and look for explanations for these problems. You may obtain assistance in Mathskeller, the Study, in recitation, in office hours of your teaching assistant or lecturer, in your study group, and by sending e-mail to your TA from mathclass.org.

The Study is located in the basement of Young Library and provides drop in tutoring in a number of subjects. The study is open in the evenings from Sunday to Thursday. Visit `http://www.uky.edu/UGS/study` for more information.

The Mathskeller is located in the basement of the Classroom building and is open from 9 to 4 Monday to Friday. Instructors in MA 113 will hold some of their office hours in Mathskeller. In addition, assistance with `mathclass.org` will be available and students may print out their assignments from `mathclass.org` in the Mathskeller.

- Be sure to review carefully every item on the review homework assignments.
- Before each exam, take a practice exam. Exams from past semesters are available at `http://www.ms.uky.edu/~rbrown/courses/exams-113`. Note that there may be small changes in the syllabus from semester to semester.
- Work problems. Work more problems. Many students choose to work additional problems above the required assignments. The following sources of additional problems are available. 1. The textbook (see the course calendar). 2. The common version of web homework problems. 3. Additional versions of your web homework. Choose your favorite number to use as the version number at `mathclass.org`. 4. The warmup homework sets, W1-W4 at `mathclass.org`.
- Did we mention that you should go to class?

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COMMON ERRORS.

The following errors are commonly found in mathematics homework and exams. Please try to avoid these mistakes. When grading papers, the three letter abbreviations may be used to indicate these errors.

- EQN. Misuse of the equal sign. When we write the equal sign, $=$, the quantities appearing on either side should be equal. It is common for students to write something like $x^2 = 2x$, when they mean $\frac{d}{dx}x^2 = 2x$.
- ALG. Mistakes in algebra. It is not true that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$. Try $a = b = 1$. It is also not correct cancel b^2 to obtain $\frac{a+b^2}{b^2} = a$.
- EXP. Incomplete explanations. Your goal in working a problem in mathematics is not just to arrive at a correct answer, but to also show that you understand why the answer is true. In general, your explanations should be written out in complete sentences.
- UNI. Missing or incorrect units. In applied problems, one should give the units for the answer. Thus the velocity of a wombat might be 10 meters/second or 1000 centimeters/second, but never 10.
- AXE. Label your axes. When drawing a graph, you should give the coordinate for each axis and label a few tick-marks on each axis.

Worksheet on induction MA113
Calculus I Fall 2006

First, let us explain the use of $\sum$ for summation. The notation

$$\sum_{k=1}^{n} f(k)$$

means to evaluate the function $f(k)$ at $k = 1, 2, \ldots, n$ and add up the results. In other words:

$$\sum_{k=1}^{n} f(k) = f(1) + f(2) + \ldots + f(n).$$

For example:

$$\sum_{k=1}^{4} k^2 = 1 + 4 + 9 + 16,$$

$$\sum_{k=1}^{n} (2k - 1) = 1 + 3 + 5 + \ldots + 2n - 1,$$

and

$$\sum_{k=3}^{2n} 1 = 2n - 2.$$

The principle of mathematical induction is used to establish the truth of a sequence of statements or formula which depend on a natural number, $n = 1, 2, \ldots$. We will use P_k to stand for a statement which depends on k. For example, P_k might stand for the statement "The number $2k - 1$ is odd." These statements are true for $k = 1, 2, \ldots$

The principle of mathematical induction is:

Principle of mathematical induction. Suppose that P_n is a sequence of statements depending on a natural number $n = 1, 2, \ldots$. If we show that:

- P_1 is true
- For $N = 1, 2, \ldots$: If P_N is true, then P_{N+1} is true.

Then, we may conclude that all the statements P_n are true for $n = 1, 2, \ldots$.

To see why this principle makes sense, suppose that we know P_1 is true, then the second step allows us to conclude P_2 is true. Now that we know P_2 is true, the second step allows us to conclude P_3 is true. If we repeat this $n - 1$ times, we conclude that P_n is true.

This principle is useful because it allows us to prove an infinite number of statements are true in just two easy steps! We usually call the first step the *base case* and the second step is called the *induction step.*

Below are several examples to illustrate how to use this principle. The statement P_N that we assume to hold is called the *induction hypothesis.* The key point in the induction step is to show how the truth of the induction hypothesis, P_N, leads to the truth of P_{N+1}.

Example 1. Show that for $n = 1, 2, 3, \ldots$, the number $n^2 - n$ is even.
Solution. Base case. This is easy. If $n = 1$, then $n^2 - n = 1^2 - 1 = 0$ and 0 is even.
Induction step. We suppose that $N^2 - N$ is even and we want to use this assumption to show that $(N+1)^2 - (N+1)$ is even. We write $(N+1)^2 - (N+1) = N^2 + 2N + 1 - N - 1 = N^2 + N + 2N$. Now $2N$ is even when N is a whole number and $N^2 + N$ is even by our induction hypothesis. As the sum of two even numbers is again even, we conclude that $(N+1)^2 + (N+1)$ is even. ∎

Example 2. Show that for $n = 1, 2, \ldots$, we have

$$\sum_{j=1}^{n} 2j = n(n+1).$$

Solution Base case. If $n = 1$, then $n(n+1) = 1 \cdot 2 = 2$. Also,

$$\sum_{j=1}^{1} 2j = 2.$$

Thus both sides are equal if $n = 1$.
Induction step. Now suppose that the formula $\sum_{j=1}^{N} 2j = N(N+1)$ is true and consider the sum

$$\sum_{j=1}^{N+1} 2j = \sum_{j=1}^{N} 2j + 2(N+1).$$

We use our induction hypothesis that $\sum_{j=1}^{N} 2j = N(N+1)$ to conclude that

$$\sum_{j=1}^{N+1} 2j = N(N+1) + 2(N+1).$$

Simplifying this last expression gives

$$N(N+1) + 2(N+1) = N^2 + N + 2N + 2 = N^2 + 3N + 2 = (N+2)(N+1).$$

Since $(N+2)(N+1) = (N+1+1)(N+1)$, we have shown that the formula

$$\sum_{j=1}^{N+1} 2j = (N+1+1)(N+1)$$

is true. This completes the induction step and thus the proof by induction. ∎

Example 3. All horses are the same color.

Solution. We will show by induction that any set of N horses consists of horses of the same color.
The base case is easy. If we have a set with one horse, then all horses in the set are the same color.
We assume as our induction hypothesis that any set of N horses consists of horses of the same color. We take a set of $N+1$ horses, and put one of the horses in the barn for a moment. By our induction hypothesis, the remaining N horses are all of the same color. Now, we put a different horse in the barn. Again, the remaining N horses are all the same color. It follows that the set of $N+1$ horses are all the same color. ∎

Below is a selection of problems related to mathematical induction. You should begin working on these problems in recitation. Write up your solutions carefully, elegantly, and in complete sentences.

1. (a) For $n = 1, 2, 3, 4$, compute
$$\sum_{k=1}^{n}(2k-1).$$
Make a guess for the value of this sum for $n = 1, 2, \ldots$.
   (b) Use mathematical induction to prove that your guess is correct.

2. Use the principle of mathematical induction to prove that
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

3. Define $n! = n(n-1)!$ and $0! = 1$. Thus, $n! = n(n-1)\ldots 1$, if $n = 1, 2, 3, 4, \ldots$.
   (a) Define $A_{n,k} = \frac{n!}{(n-k)!k!}$. Show that we have the formula
$$A_{n+1,k+1} = A_{n,k} + A_{n,k+1}.$$
Hint: Obtain a common denominator on the left-hand side. Do not use the principle mathematical induction to prove this.
   (b) For $n = 1, 2, \ldots$, show that there are numbers $B_{n,k}$ so that
$$(x+y)^n = \sum_{k=0}^{n} B_{n,k}x^k y^{n-k}.$$
You should be able to do this by induction on n. For the base case, write down $B_{1,0}$ and $B_{1,1}$.
In the induction step, you should write $(x+y)^{N+1} = (x+1)(x+y)^N = (x+y)(B_{N,0}x^0y^N + B_{N,1}x^1y^{N-1} + \ldots + B_{N,N}x^Ny^0)$ and discover a formula that expresses $B_{N+1,k}$ in terms of some of the numbers $B_{N,j}$, $j = 0, \ldots, N$ which are known from the induction hypothesis.

(c) Is there a relation between the numbers $A_{n,k}$ and $B_{n,k}$? (A proof is not expected.)

Additional problems. Below are some additional exercises for you to consider. You will not be able to solve all of these problems at this time. These problems will not be collected.

1. Find the flaw in the proof that all horses are the same color.

2. Let $f_1(x) = x - 2$ and then define f_n for $n = 1, 2, \ldots$ by $f_{n+1}(x) = f_1(f_n(x))$. (It is the principle of mathematical induction which tells us that these two statements suffice to define f_n for all $n = 1, 2, 3, \ldots$.) Use mathematical induction to prove that
$$f_n(x) = x - 2n.$$

3. Let P_n be the statement: $n^2 - n$ is an odd integer.

   (a) Show that if P_n is true, then P_{n+1} is true.

   (b) Is P_1 true?

   (c) Is P_n true for any n?

4. Let $f(x) = \sin(2x)$. Prove that for $n = 1, 2, \ldots,$
$$\frac{d^{2n}}{dx^{2n}} f(x) = (-4)^{2n} \sin(2x).$$

5. Prove that
$$\frac{d}{dx} x^n = nx^{n-1}, \qquad n = 1, 2 \ldots.$$
Hint: For the base case $n = 1$, use the definition of the derivative. For the induction step write $x^{n+1} = x \cdot x^n$ and use the product rule.

6. Prove that
$$\frac{d}{dx} \frac{1}{x^n} = \frac{-n}{x^{n+1}}, \qquad n = 1, 2 \ldots.$$

7. Prove that
$$\frac{d^n}{dx^n} x^n = n!, \qquad n = 0, 1, \ldots$$

8. (a) Find a simple formula for
$$\sum_{k=1}^{n} ((k+1)^2 - k^2) = 2^2 - 1 + (3^2 - 2^2) + \ldots + n^2 - (n-1)^2 + (n+1)^2 - n^2.$$

(b) Using your answer to part a), find a simple expression for

$$\sum_{k=1}^{n}(2k-1).$$

To do this you should simplify each summand on the left.

9. Use mathematical induction to prove that

$$\sum_{j=1}^{n} j^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

August 16, 2006

Worksheet on the definition of a derivative. MA113
Calculus I Fall 2006

This worksheet is designed to provide practice in using the definition of the derivative to find the slope of a tangent line. Some of the calculations are sometimes a bit tedious. However, they provide good practice in algebra. More importantly, we should learn the basics (nothing in calculus is more basic than the definition of the derivative) well, before we move on to more elegant approaches.
As always, your work should be written out neatly and carefully. Use complete sentences.
For these exercises, it will be useful to recall that we may remove radicals from the denominator of an expression by multiplying by the conjugate:

$$\frac{1}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{a}+\sqrt{b}}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}.$$

1. Write the definition of the derivative of a function f at a point x.

2. Use the definition of the derivative to find the derivative of $f(x) = \sqrt{2-3x}$. What is the domain of f? What is the domain of f'?

3. Suppose (x, y) is a point on a circle centered at the origin. Using well-forgotten facts from geometry, find the slope of the tangent line to the circle which passes through (x, y). State clearly the fact from geometry that you use to find the slope.

4. Now consider the unit circle, the circle of radius one which is centered at the origin. The part of the unit circle that lies above the x-axis is give by the graph of the function $g(x) = \sqrt{1-x^2}$. Use the definition of the derivative to find the slope of the tangent line to the unit circle at the point $(x, \sqrt{1-x^2})$.

5. Do your answers to parts 3 and 4 agree?

August 15, 2006

Calculus I *Fall 2006*
MA113

Duties of teaching assistants.

1. In general, a recitation should consist of:
   (a) A short "lecture" to review material and work examples. Examples will generally be drawn from the common homework sets at WHS.
   (b) A period of group work devoted to the homework set or worksheet of the day and additional activities as you deem appropriate.
   (c) Practice quizzes as in the course schedule.
2. Make a brief syllabus which gives your name, office hours and contact information. Hand this out on the first day.
3. Grade pretests and return to the lecturer to be given to the student the next day. The scores on these tests may help to identify one or two students per section who have registered for calculus but are clearly not prepared. This applies to students who score 0-4. Students in the range 4.5-7.5 should be warned to seek out extra practice on routine skills. Students who score 8 and above are prepared, but should not plan to coast through the semester.
4. Teaching assistants must take attendance in recitation in order to assign MA 193 grades. Accept any reasonable excuse for an excused absence. However, the student must make the effort to explain his or her absence.
5. Teaching assistants will be responsible for recording grades. I believe that we can use webclass for this. Be sure to download and/or print a copy of grades in the unlikely event that the servers melt down. Lecturers can access grades through webclass. However, please ask to find out if the lecturer(s) that you are working with would prefer a paper copy of grades. (Warning: This is new version of webclass. I am not yet familiar with all the details of the gradebook.)
6. Prepare practice quizzes and distribute a copy of the quiz to each teaching assistant and lecturer by 3:30pm the day before the quiz is to be given. A schedule will be prepared giving assignments for preparing quizzes.
7. Teaching assistants should schedule one hour in Mathskeller and two additional office hours in Mathskeller or your office.
8. Teaching assistants will help with proctoring and grading of exams and will do most of the grading of the worksheets.

9. Worksheets should be graded within one week. No worksheet should be handed in before an exam and returned after an exam.

10. Exams will be graded Wednesday during the day. Everyone should complete their grading assignment by 5pm Wednesday.

11. Teaching assistants should provide a report to the coordinator on the scores for each exam. Please submit these reports by e-mail before 10am on the Thursday following a mid-term exam. For each section, this report should include three items:

    - The number of papers scoring above 30.
    - The mean score of the papers scoring above 30.
    - The number of papers scoring 30 or below.

    Papers scoring 30 or below will be discarded in preparing a course average.

    In addition, each lecturer may want additional information about their section.

12. Teaching assistants should attend weekly instructor meetings. These meetings will be at 3pm on Wednesday in a place to be determined.

13. If a student complains about a grade, ask that the student write down the question and hand it in with the paper. Bring the paper to your lecturer and to review the grading and make changes if necessary.

14. Answer e-mail from within mathclass.org. Please let the course coordinator know if you run across a problem that appears to be incorrectly set.

15. Monitor students who have applied to join the course through mathclass.org and admit these students.

    After receiving approval from the instructor, students may drop a class at mathclass.org. In general, there is no need for a student to drop from the web homework system. There is a small danger that one might drop the wrong student and eliminate important records.

    Students may switch sections at mathclass.org by adding the new section and dropping the old section. Students should use the same account, so that their record of completed assignments will be visible in the new section. If a student asks about changing sections, emphasize that they should add a new section, but that they should not create a new account.

16. The new mathclass.org seems to allow each class to have different due dates for an assignment. Do not take advantage of this feature. If one section has a later due date, then students may join a section with an earlier due date, find the answers and submit these answers for credit.

If there is a need to adjust a due date, please contact Brown and ask that the due date be changed for all sections.

17. Prepare gradesheets and give to lecturer for signature.

Duties of lecturers.

1. Make a brief syllabus which gives your name, office hours and contact information. Hand this out on the first day.
2. Deliver brilliant, insightful lectures exposing the beauty and power of the Calculus.
3. Write exams.
4. Supervise writing, invigilating and grading of exams. We will take turns as "exam tzar". A schedule will be distributed with exam writing and exam tzar assignments.
5. The exam csar should: Prepare a key with point assignments for problems. Provide Brown with a clean copy of the key to scan and post on the web. Answer questions from graders. Review a few exams from each teaching assistant to make sure that the key is being followed and that following the key is producing sensible results.
6. Each lecturer will handle appeals on grading in their section. Report to Brown or exam czar if there appear to be serious, systematic mistakes in grading.
7. Review problem sets and make suggestions for problems to be edited, deleted, or added.
8. Review practice quizzes written by your teaching assistants.
9. Take turns leading weekly meetings.
10. Participate in setting any curves on exams. Our goal is produce a course average by 12noon on Thursday after each midterm exam. This information will be circulated by e-mail and each instructor will submit their suggestion for a curve.
11. Keep a record of excused absences that caused a student to miss a homework assignment. I recommend that students who are excused from homework have their homework grade computed as an average of the assignments they submit.
12. Work with teaching assistants to administer and grade makeup exams.
13. Visit recitations to help TA's develop as teachers. Follow up if serious problems are identified.
14. Review midterm and final grades and sign gradesheets. My target is to submit midterm grades on 9 or 10 October.

Duties of coordinator.

1. Take credit for our successes, observe that any failures are the fault of the high schools.
2. Adjust due dates for homework, if needed.
3. Prepare schedules for exam tsars, writing exams, and writing practice quizzes.
4. Edit web homework.
5. Prepare makeup exams.

Calculus I *Fall 2006*
MA113

The following is an outline for our first recitation, Thursday, 24 August 2006.

- Welcome activity. (10 minutes) Collect names of students, other information if desired. Hand out teaching assistant contact information.
- Review selected material for review homework set, A1. (20 minutes) One should not expect to cover everything in the review set. Consider working one or more of the following examples.
  - Describing how to transform the graph of f to obtain the graph of $g(x) = f(ax + b)$.
  - Find the equation of a line. Be sure that students are familiar with the "point-slope" form for the equation of a line as well as the "slope-intercept" form.
  - Find a quadratic polynomial given the roots and one additional value. (Hint: Write the polynomial as $A(x - r_1)(x - r_2)$ where r_1 and r_2 are the roots.)
- Work in groups on homework, answer questions as necessary. (15 minutes) (In theory, at the first lecture, the lecturer should have asked each student to register at mathclass.org, print out their assignment A1 and bring it to recitation.)
- Administer pretest. (30 minutes) Allow students to leave when finished.

Pretest for Calculus I

On a separate sheet of paper, answer the following questions and show your work. Put your name and section number on your answer sheet. Allow 30 minutes for this pretest. A calculator may not be used in answering these questions.

This pretest is to help you determine if you are ready to begin the study of calculus. Your score on this pretest will not be counted toward your grade in calculus.

If you are not able to answer 8 of these questions correctly, then you may have difficulty with Calculus I. The previous sentence is not equivalent to the statement, "If you are able to answer 8 of these questions correctly, then you will not have difficulty with Calculus I"

1. Write as a single rational expression
$$\frac{1}{2x+1} + \frac{1}{2x-1}.$$
2. If the hypotenuse of a right triangle has length 4 and one of the other sides has length 3, find the length of the remaining side.
3. Factor the numerator and denominator and cancel common factors.
$$\frac{\cos x \sin x + \cos^2 x}{\cos^2 x}.$$
4. Factor the numerator and denominator and cancel common factors.
$$\frac{x^2 - 3x}{x^2 - 9}.$$
5. Find the exact solution(s) of $x^2 + 3x = 10$.
6. Find the exact solution(s) of $x^2 = 3x - 1$.
7. Solve
$$\frac{2}{3}(x-1) + \frac{1}{2}(x+4) = 2.$$
8. Find the equation of the line that passes through the points $(x, y) = (2, 4)$ and $(x, y) = (-1, 9)$. Simplify to put your answer in the form $y = mx + b$.
9. If Lola runs at the speed of 8 feet/second, give her speed in yards/minute.

   Recall that there are 60 seconds in a minute and 3 feet in a yard.
10. Write in the form $x^a y^b$.
$$\frac{x^2 y^3}{(x^4 y)^2}$$

Solutions to pretest for Calculus I

Give one point per problem. Generally, problems should be marked right or wrong. You may award 1/2 point for students who solve a problem correctly but make a minor mistake in arithmetic. Please do not duplicate these answers and distribute them to students.

1. Write as a single rational expression

$$\frac{1}{2x+1}+\frac{1}{2x-1}.$$

Answer: $\frac{4x}{4x^2-1}$

2. If the hypotenuse of a right triangle has length 4 and one of the other sides has length 3, find the length of the remaining side.

Answer: $\sqrt{7}$

3. Factor the numerator and denominator and cancel common factors.

$$\frac{\cos x \sin x+\cos^2 x}{\cos^2 x}.$$

Answer: $\frac{\sin x+\cos x}{\cos x}$

4. Factor the numerator and denominator and cancel common factors.

$$\frac{x^2-3x}{x^2-9}.$$

Answer: $\frac{x}{x+3}$

5. Find the exact solution(s) of $x^2+3x=10$.

Answer: $-5,\ 2$

6. Find the exact solution(s) of $x^2=3x-1$.

Answer: $\frac{3\pm\sqrt{5}}{2}$

7. Solve

$$\frac{2}{3}(x-1)+\frac{1}{2}(x+4)=2.$$

Answer: 4/7

8. Find the equation of the line that passes through the points $(x,y)=(2,4)$ and $(x,y)=(-1,9)$. Simplify to put your answer in the form $y=mx+b$.

Answer: $y=\frac{-5}{3}x+\frac{22}{3}$

9. If Lola runs at 16 feet/second, give her speed in yards/minute.
   Recall that there are 60 seconds in a minute and 3 feet in a yard.
   Answer: 320 yards/minute

10. Write in the form $x^a y^b$.

$$\frac{x^2y^5}{(x^2y)^3}$$

    Answer: $x^{-4}y^2$ or y^2/x^4

To: Ma113 instructors.
From: Russell Brown
Date: 14 August 2006

Subject: Pretest guidelines

I recommend that students who score 8 or above on the pretest be encouraged to continue in Calculus. Students who score 4 or below should be discouraged from continuing in MA 113. Students who score between 4 and 8 should be warned that there are deficiencies in their background and if they continue in MA 113 they should plan on doing extra work to succeed in this course. These recommendations should come from the lecturer, rather than a teaching assistant.

The table below summarizes the performance in MA113, Calculus I, of a number of students who have taken the pretest. Success is defined to mean earning A, B or C. The information comes from 249 students in 14 sections from three different semesters and three different instructors. Note that this table only includes students who finish the course with ABCDEW.

Score	Fraction of class	Probability of success
10	10%	88%
9-9.5	19%	79%
8-8.5	17%	69%
7-7.5	16%	80%
6-6.5	9%	52%
5-5.5	11%	50%
4-4.5	9%	18%
3-3.5	5%	46%
0-2.5	3%	14%

The second table gives data from fall 2005 and gives the distribution of scores of all students who took the pretest. According to my guidelines, 50% to 60% of the students are qualified to continue in this course.

Score	Cumulati fraction
10	0.1
9-9.5	0.32
8-8.5	0.47
7-7.5	0.57
6-6.5	0.78
5-5.5	0.83
4-4.5	0.9
3-3.5	0.98
0-2.5	1