Package guide

The attached packet consists of three parts. Student packets 1 and 2 have been duplicated. Copies of student packet 1 will be given to the lecturers to distribute to students on Wednesday, Jan. 9. Copies of student packet 2 will be given to the teaching assistants to distribute to students on Thursday, Jan. 10.

Student packet 1:

(1) Syllabus
(2) Calendar
(3) Some suggestions on how to study mathematics
(4) Common errors
(5) Handout on induction
(6) Worksheet 1

Instructor information:

(1) Outline of TA and lecturer duties
(2) Suggestion for first recitation
(3) Pretest, its solution, and pretest guidelines

Student packet 2:

(1) Pretest

Syllabus for MA 113 - Calculus I, Spring 2008

Web site

The home page for this course is at

`http://www.ms.uky.edu/~uwenagel/CALC-I-s08/113-home.`

It is designed to help you and to provide information. In particular, handouts and solutions to exams or written assignments will be posted at this web site.

Schedule

- Lectures: MWF
- Recitations: TR
- Exams: There are three uniform midterm exams and one final exam. The final exam will be cumulative though with an emphasis on the material covered since the third exam.
  - Exam 1: Room TBA, Feb. 5, 7:30 - 9:30 pm
  - Exam 2: Room TBA, Mar. 4, 7:30 - 9:30 pm
  - Exam 3: Room TBA, Apr. 8, 7:30 - 9:30 pm
  - Final exam: Room TBA, May 1, 8:30 - 10:30 pm

Textbook

Calculus (5th edition) by James Stewart, ISBN 0-534-39339-X

Material and goals

In Calculus I, we will learn about derivatives, integrals and the fundamental theorems of calculus. We begin by introducing the notion of a limit. Limits are essential to defining derivatives and integrals. By the end of the semester students should know precise definitions of the derivative and the integral and the fundamental theorem of calculus which gives the relation between the derivative and the integral. We will illustrate the methods and ideas of calculus by studying several physical and geometric problems. We will study the interpretation of the derivative as velocity or slope of a tangent line, the trajectory of a body falling under the influence of gravity, the interpretation of the integral as area or distance traveled and the use of the integral in computing volumes of familiar solids such as a sphere or a cone.

We will cover most of Chapters 1 to 6 of Stewart's book. Please see the course calendar for a detailed listing of sections.

Exposure to the precision needed in Calculus will foster critical thinking and rational reasoning. In order to help you learn to articulate mathematical ideas there will be six written assignments. Your solutions to these assignments are expected to be carefully drafted documents that are written up in complete sentences. You should lay out and explain all the arguments you used to arrive at the solution.

Grading

You can earn up to 500 points in the course based on the following activities:

3 exams	300 (100 points each)
Final exam	100
Homework and attendance	100
Total	**500**

The 100 points for homework and attendance are computed based on the following components:

Web homework	95 points
Written assignments	60 points (10 points each)
Attendance of the lectures	45 points
Total divided by 2 (and rounded up)	**100 points**

Your course grade will be based on the number of points you earn according to the following scheme:

Total earned course points (out of 500)	450-500	400-449	350-399	300-349	0-299
Final **course grade**	A	B	C	D	E

Homework and Quizzes

There are three types of homework, only the first two count towards the grade.

(1) The bulk of the homework will be completed using the well tested web-based homework system that grades your solutions and records your scores. You find it at
`http://www.mathclass.org` (see below for administrative details on using this website).

We recommend to approach the web-based homework assignments via the following rules.

- Start to work on an assignment as soon as the corresponding material is discussed in class.
- Print out copies of your personal and of the common assignments (it is free in the Mathskeller, the student staff will show you how to do so) and put them in a notebook.
- Get together with classmates to work on the problems via the printouts.
- Write down the solutions in your notebook and only thereafter enter your solutions on the webpage. Only correct solutions to your **personal version** of the homework assignment give you credit! Notice that for each web-based homework problem **you may resubmit your answer as often as you wish before the due date!** Only your final (and hopefully correct answer) will be recorded for your homework grade. You may find your score at `http://www.mathclass.org` by clicking homework scores on the main page.
- Bring the notebook with you when going to office hours.
- Bring copies of the common problems to recitations. They will be discussed there.
- You are encouraged to discuss homework problems and the course material with each other. However, when it comes time for you to write up or enter the solutions, we expect you to do this completely on your own. It would be the best for your understanding if you put aside your notes from the discussions with your classmates and wrote up the solutions entirely from scratch.

If you feel you have worked a problem correctly and WHS marks it incorrect, please contact your teaching assistant or lecturer, for example, by e-mail.

(2) In addition to the web homework, there will be six written assignments, worksheets, that will be graded by humans. Your solutions will be graded for mathematical correctness, and for clarity of exposition. Students who wish to receive full credit should write in complete sentences laying out the arguments carefully.

(3) There are various optional homework problems that do not count towards your grade: web-based Warmup and Review assignments as well as assignments from the textbook. All these problems are listed in the course calendar.

The optional assignment A0 is intended to introduce you to the syntax to enter mathematical expressions in the web homework system. The review assignments AR, BR, CR, and DR are study guides for each exam. All students are strongly advised to complete these review assignments.

Quizzes will be given regularly during recitations (see the course calendar). The quizzes will not be graded. They should help you to cope with a test situation where you have to work the given problems with closed books and a limited amount of time.

Attendance

You are expected and strongly advised to attend all lectures and recitations.

Lecturers will take attendance in all lectures beginning January 11. Your score is based on the percentage of lectures you attend. You will receive full credit (45 points, see above) if you have at most 2 unexcused absences.

MA 193

In addition to the 4 hours of credit for MA 113, the department offers one additional hour of credit as MA 193 on a pass/fail basis. You will pass MA 193 if you have at most 2 unexcused absences during MA 113 recitations and you pass MA 113. If you fail MA 113 or have 3 or more unexcused absences you will fail MA 193.

Your section number for MA 193 equals your section number for MA 113. If you drop or change sections of MA 113, please make sure to also drop or change sections of MA 193.

Getting help

If you are having difficulty with any aspect of the course, you should seek help immediately.

If you are having trouble with a homework problem, you can send an e-mail through the online homework system to your teaching assistant. Try to provide as much information as possible in your help request. For example, you should at least describe how you attempted the problem and at least guess where you might be going wrong.

If you need more help than what can be provided by the online help, meet with your instructor or teaching assistant. They will be happy to help.

You can also seek help in the **Mathskeller** that is located in room CB 065 in the basement of the classroom building. Many instructors and teaching assistants from the Department of Mathematics will hold office hours in the Mathskeller. In addition, limited drop-in tutoring is available. You can seek help from any of the instructors or teaching assistants - not just your own. The Mathskeller is open from 9 am to 5 pm Monday through Friday (except academic holidays) during the semester. Additional information is available at `www.mathskeller.org`.

Furthermore, you can seek help in **The Study** located on the 3rd floor of the Commons, South Campus. Free math tutoring is available Su-Th 5-8 pm. Just come by The Study on the 3rd floor of the Commons. Academic Enhancement also provides drop-in peer tutoring by experienced undergraduate students who have successfully navigated the courses for which they tutor. A regular schedule of all tutoring is available on The Study's website at `www.uky.edu/ugs/study`. You can also call 257-1356.

Using the web homework system

Students enrolled in MA 113 will have an account created for them. **Please do not create an account unless asked to do so by your instructor.** If you create an account, you will not be able to add your class to this account.

There are three methods to login to your account:

(1) *Active directory logins:* The preferred method for logins to `mathclass.org` is with the UK Active Directory user name and password. This is also the user name and password that are used to access other systems including `myuk.uky.edu` and `exchange.uky.edu`. Thus, if your user name is `skova01`, you will enter `ad\skova01` as the user name and then the password for your Active Directory account. Note that `mathclass.org` will require you to include the prefix `ad\` while other sites on campus may not. Students who are in the Medical Center domain should use the prefix `mc\`.

(2) *Student identification number and mathclass.org password:* Students may also log in to their account at `mathclass.org` using their eight digit student identification number as a user name and a password that is local to `mathclass.org`. The initial password will be `u$654321` where `654321` are the last six digits of your student identification number. The student identification cards have a nine digit number that always begins with a 9. The student identification number that we use are the eight digits which appear after the "9." Most student identification numbers will begin with a 1.

(3) You may also use the e-mail address for your account as user name. To find this address, visit the link `Don't know which User Name or e-mail to use?`. This link will also allow you to look up your Active Directory user name.

Warning: You may have a different password for each login method. However, both methods give you access to the same account.

Many more details on using your account at mathclass.org can be found at

`http://www.math.uky.edu/~rbrown/whs/mathclass.pdf`.

Late homework

No late submissions of web homework will be accepted. If an emergency or illness takes you away from school, please meet with your lecturer to discuss your situation and ask to be excused from an assignment, if appropriate. If you have a scheduled absence (travel or authorized university absence) you must still submit the web homework by the deadline.

Written assignments are due at the beginning of the lecture. If an emergency or unexpected absence prevents you from turning in the assignment, please see your lecturer to request permission to turn in the assignment late. If you have a scheduled absence (travel or authorized university absence) you should arrange to turn in the paper before leaving school. Unexcused and late submissions will be penalized 10% if the paper is turned in late on the due date and an additional 20% for each day that it is late.

Calculators

Students may use a graphing calculator on exams and homework. The use of machines with symbolic manipulation capabilities is not allowed during examinations. Thus, no TI-89's, TI-92's, no HP-48's or laptop computers may be used on exams. Please talk to your lecturer if you have any questions as to whether a particular machine may be used on a test. We may clear the memory of calculators before or during an examination.

Academic Honesty

Students are encouraged to work together to understand a problem and to develop a solution. However, the solution you submit for credit must be your own work. In particular, you should write your written solutions independently. Copying on exams and usage of books or notes during examinations is not allowed. Cheating or plagiarism is a serious offense, and it will not be tolerated.

Calendar for MA 113 - Calculus I

Spring 2008

Lecture / Recitation	Section, topic, assignments	Assignments (textbook problems are optional)
9-Jan	Ch. 1 Review of functions	Review, p. 56 #1,2,3,5,6,8-12,16-19, A0: Syntax (optional)
10-Jan	Pretest, Assignment A1	A1: Review
11-Jan	Mathematical induction, handout.	
14-Jan	2.1 Tangent and velocity problems	§2.1 #1,2,3,5,6,8,9
15-Jan	Worksheet 1	
16-Jan	2.2 Limit of a function 2.4 The precise definition of a limit (lightly)	§2.2, #2,4,5,6,9,12,13,15,25,28
17-Jan	Assignment A2, practice quiz 1	A2: Tangents and velocity
18-Jan	2.3 Calculating limits using the limit laws	§2.3 #1,2,11,13,15,17,20,22,28,39,40,49
21-Jan	Martin Luther King Day, academic holiday	
22-Jan	Assignment A3 12m submission deadline for A1 and A2	A3: Limits
23-Jan	2.5 Continuity Worksheet 1 due in class.	§2.5, #1,3,4,5,6,7,9,11,17,21,37
24-Jan	Assignment A4, worksheet 2, practice quiz 2	A4: Continuity
25-Jan	2.6 Tangents, velocities and rates of change	§2.6 #1,2,3,5,13,15,17,18,23
28-Jan	3.1 Derivatives 12m submission deadline for A3 and A4.	§3.1 #3,4,6,7,9,12,15,16,19,22,25,26
29-Jan	Assignment A5	A5: Tangents, secants, difference quotients
30-Jan	3.2 The derivative as a function Worksheet 2 due in class Last day to drop	§3.2 #1,2,4,7,10,12,17,25,36,39,41
31-Jan	Assignment A6, practice quiz 3	A6: Introduction to the derivative
1-Feb	Review	AW: Warmup exercises (optional)
4-Feb	Review 12m submission deadline for A5 and A6.	
5-Feb	Assignment AR (not graded) **Exam 1**, 7:30-9:30pm, room TBA.	AR: Review for Exam 1
6-Feb	3.3 Differentiation formula	§3.3 #5,10,16,18,21,25,28,33,40,44,53,57,58,62
7-Feb	Assignment B1	B1: Differentiation rules
8-Feb	Appendix D, Trigonometry	Appendix D, #1,4,7,10,13,15,23,26,29,30,31
11-Feb	3.5 Derivatives of trigonometric functions 12m submission deadline for B1.	§3.5 #3,6,9,12,18,29,30,35,36,43
12-Feb	Assignment B2, worksheet 3	B2: Review of trigonometry
13-Feb	3.6 The chain rule	§3.6 #1,5,6,7,10,15,16,19,25,28,45,46,55,56
14-Feb	Assignment B3, practice quiz 4	B3: Differentiation of trigonometric functions
15-Feb	3.7 Implicit differentiation	§3.7 #3,4,7,10,14,15,26,29,35,39
18-Feb	3.8 Higher derivatives 12m submission deadline for B2 and B3.	§3.8 #1-3,11,18,25,26,39,41,44,49,50,53
19-Feb	Assignment B4	B4: The chain rule
20-Feb	3.9 Related rates Worksheet 3 due in class.	§3.9 #1,2,4,6-8,10-12,14-17,20-22
21-Feb	Assignment B5, worksheet 4, practice quiz 5	B5: Implicit differentiation, higher order derivatives
22-Feb	3.9 Related rates, continued	
25-Feb	3.10 Linear approximation (no differentials) 12m submission deadline for B4 and B5.	§3.10 #1,3,7,8,13,15,27,31,32,37
26-Feb	Assignment B6.	B6: Related rates
27-Feb	4.9 Newton's method Worksheet 4 due in class.	§4.9 #1,4,5,6,11,27,31,34,35
28-Feb	Assignment B7, practice quiz 6	B7: Linear approximation, Newton's method
29-Feb	Review	BW: Warmup exercises (optional)
3-Mar	Review 12m submission deadline for B6 and B7.	
4-Mar	Assignment BR (not graded) **Exam 2**, 7:30-9:30 pm, room TBA	BR: Review for Exam 2

Calendar for MA 113 - Calculus I

Spring 2008

Lecture / Recitation	Section, topic, assignments	Assignments (textbook problems are optional)
5-Mar	4.1 Maximum and minimum values	§4.1 #1,2,3,4,5,9,11,17,18,23,47,48,52
6-Mar	Assignment C1	C1: Extreme values
7-Mar	4.2 The mean value theorem Last day to withdraw	§4.2 #1,3,5-8,15-19,22
10 - 15 Mar	Spring Break	
17-Mar	4.3 Derivatives and the shape of a graph 12m submission deadline for C1.	§4.3 #1,2,5,6,7-9,11-17,22-26,29,31,33
18-Mar	Assignment C2, worksheet 5	C2: The mean value theorem
19-Mar	4.4 Limits at infinity	§4.4 #1-4,9,11,13,15,17,19,21,23,35,37,39,43,58
20-Mar	Assignment C3, practice quiz 7	C3: Derivatives and the shape of a graph
21-Mar	4.5 Summary of curve sketching	§4.5 #3,12,13,17,23,27,31
24-Mar	4.7 Optimization problems 12m submission deadline for C2 and C3.	§4.7 #2,3,6,7,10,16,19,22,29,32,35,51,52.
25-Mar	Assignment C4	C4: Limits at infinity and curve sketching
26-Mar	4.7 continued Worksheet 5 due in class	
27-Mar	Assignment C5, worksheet 6, practice quiz 8	C5: Optimization
28-Mar	4.10 Anti-derivatives	§4.10 #1,3,5,7,21,23,25,31,36,37,39,40,53,55,68,71
31-Mar	5.1 Areas and distances 12m submission deadline for C4 and C5.	§5.1 #1,3,4,5,11,12,20,22
1-Apr	Assignment C6	C6: Anti-derivatives
2-Apr	5.2 The definite integral Worksheet 6 due in class.	§5.2 #1,7,9,17,19,25,29,30,33-36,39,47-49,55,57
3-Apr	Assignment C7, practice quiz 9	C7: The definite integral and area
4-Apr	Review	CW: Warmup exercises (optional)
7-Apr	Review 12m submission deadline for C6 and C7	
8-Apr	Assignment CR (not graded) **Exam 3**, 7:30pm-9:30pm, room TBA	CR: Review for Exam 3
9-Apr	5.3 The fundamental theorem of calculus	§5.3 #1,7-11,13,19,21,23,25,27,31,33,51,
10-Apr	Assignment D1	D1: The fundamental theorem of calculus
11-Apr	5.4 Indefinite integrals	§5.4 #1,3,17,19,21,23,25,31,33,43,46,48
14-Apr	5.5 Substitution 12m submission deadline for D1	§5.5 #1,3,9,11,13,15,17,19,21,27,37,39,45,49,57,58
15-Apr	Assignment D2	D2: Indefinite integrals and substitution
16-Apr	6.1 Areas between curves	§6.1 #1,2,5,7,11,13,21,22,24,45
17-Apr	Assignment D3, practice quiz 10 12m submission deadline D2	D3: Area
18-Apr	6.2 Volume	§6.2 #1,3,12,13,14,47,48,49,53
21-Apr	Review 12m submission deadline for D3	DW: Warmup exercises (optional)
22-Apr	Assignment D4	
23-Apr	Review 12m submission deadline for D4	
24-Apr	Assignment DR (not graded)	DR: Review for Final exam
25-Apr	Review	
1-May	**Final exam**, 8:30-10:30 pm, room TBA	

Some suggestions on how to study mathematics

- Go to class and recitation.
- Go to class and take notes. The notes you produce will provide a written record of what your instructor believes to be the most important points of the course. Your notes will provide additional examples that will be helpful as you solve problems on homework. Writing notes will help focus your attention on the mathematics being presented in class.
- Go to class when exams and written assignments are returned. Be sure to collect every assignment. Review mistakes in your work so that you will be less likely to repeat the mistake on the next assignment.
- Read the textbook before you go to class. While you may not understand every detail after reading a section in the text, you will be familiar with the main ideas and class will help to deepen your understanding.
- Write careful and complete solutions to each homework problem and keep these solutions in a notebook. We will not, in general, grade your solutions. Your solutions will be useful when you review for exams. Your solutions will be helpful when you seek assistance on a problem and need to explain how you have approached the problem.

  In your work, your goal is not just to arrive at a correct answer but to also to explain your reasoning in order to convince the reader that you understand how to solve the problem and that your solution is correct.
- Form a study group. After you go to class, work on homework with fellow students. Make sure that you are able to answer every question. After you have answered every question, make sure that you understand why your answers are correct. Discuss your reasoning with your fellow students and see if others have a better approach to solving a problem.
- Attempt homework problems immediately after material is covered in class. Mark problems that you find difficult and look for explanations for these problems. You may obtain assistance in Mathskeller, the Study, in recitation, in office hours of your teaching assistant or lecturer, in your study group, and by sending e-mail to your TA from mathclass.org.

  The Study is located in the third floor of the Commons in the Kirwan-Blanding Dormitory Complex and provides drop-in tutoring in a number of subjects. The study is open in the evenings from Sunday to Thursday. Visit
  `http://www.uky.edu/UGS/study` for more information.

The Mathskeller is located in the basement of the Classroom building and is open from 9 to 5 Monday to Friday. Instructors in MA 113 will hold some of their office hours in Mathskeller. For a schedule visit `www.mathskeller.org`. In addition, assistance with `mathclass.org` will be available and students may print out their assignments from `www.mathclass.org` in the Mathskeller.

- Be sure to review carefully every item on the review homework assignments.
- Consider taking a practice exam. Exams from past semesters are available at `http://www.ms.uky.edu/~ma113/exams/`. Note that there may be small changes in the syllabus from semester to semester.
- Work problems. Work more problems. Many students choose to work additional problems above the required assignments. The following sources of additional problems are available:

  1. The textbook (see the course calendar).
  2. The common version of web homework problems.
  3. Additional versions of your web homework. Choose your favorite number to use as the version number at `mathclass.org`.
  4. The warm-up homework sets at `mathclass.org`.

- Did we mention that you should go to class?

Some common errors

The following errors are commonly found in mathematics homework and exams. Please try to avoid these mistakes. When grading papers, the three letter abbreviations may be used to indicate these errors.

- EQN. Misuse of the equal sign. When we write the equal sign, $=$, the quantities appearing on either side should be equal. We have often seen students write something like $x^2 = 2x$, when they mean $\frac{d}{dx}x^2 = 2x$.
- ALG. Mistakes in algebra. It is not true that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$. Try $a = b = 1$. It is also not correct cancel b^2 to obtain $\frac{a+b^2}{b^2} = a$.
- EXP. Incomplete explanations. Your goal in working a problem in mathematics is not just to arrive at a correct answer, but to also show that you understand why the answer is true. In general, your explanations should be written out in complete sentences. Occasionally, students will find a value of an integral or limit by using their calculator. Such answers will generally not receive credit for lack of explanation.
- UNI. Missing or incorrect units. In applied problems, one should give the units for the answer. Thus the velocity of a wombat might be 10 meters/second or 1000 centimeters/second, but never 10.
- AXE. Label your axes. When drawing a graph, you should give the coordinate for each axis and label a few tick-marks on each axis.

Handout on induction for MA 113 - Calculus I (Spring 08)

01/09/08

Motivation for studying mathematical induction. For many students, mathematical induction is probably an unfamiliar topic. Nonetheless, this is an important topic and useful in the study of calculus. Calculus involves many new ideas. To study derivatives, we have to look at the slope between pairs of points that are arbitrarily close together. To define the integral, we have to subdivide an interval into n sub-intervals for infinitely many values of n. To fully understand these operations, we have to see why infinitely many statements are true. Mathematical induction is one way to see that infinitely many statements are true.

In mathematics, we engage in deductive reasoning. We make assumptions and deduce conclusions from these assumptions. The induction step in a proof by mathematical induction provides practice in this type of reasoning.

Finally, mathematical induction provides a framework which allows us to understand why many important results in calculus, such as the rule for the derivative of a power, are true.

Summation notation. First, we explain use of $\sum$ for summation or repeated addition. The notation

$$\sum_{k=1}^{n} f(k)$$

means to evaluate the function $f(k)$ at $k = 1, 2, \ldots, n$ and add up the results. In other words:

$$\sum_{k=1}^{n} f(k) = f(1) + f(2) + \cdots + f(n).$$

For example:

$$\sum_{k=1}^{4} k^2 = 1 + 4 + 9 + 16 = 30,$$

$$\sum_{k=1}^{n} (2k - 1) = 1 + 3 + 5 + \cdots + (2n - 1),$$

and

$$\sum_{k=3}^{2n} 1 = 2n - 2.$$

Often the following identity will be useful

$$\sum_{k=1}^{n+1} f(k) = \sum_{k=1}^{n} f(k) + f(n+1).$$

The principle of mathematical induction is used to establish the truth of a sequence of statements or formula which depend on a natural number, $n = 1, 2, \ldots$. We will use P_n to stand for a statement which depends on n. For example, P_n might stand for the statement "The number $2n - 1$ is odd." These statements are true for $n = 1, 2, \ldots$

The principle of mathematical induction is:

Principle of mathematical induction. Suppose that P_n is a sequence of statements depending on a natural number $n = 1, 2, \ldots$. If we show that:

- (Base case) P_1 is true
- (Induction step) For each natural number N: If P_N is true, then P_{N+1} is true.

Then, we may conclude that all the statements P_n are true for all positive integers $n = 1, 2, \ldots$.

To see why this principle makes sense, suppose that we know P_1 is true, then the second step allows us to conclude P_2 is true. Now that we know P_2 is true, the second step allows us to conclude P_3 is true. If we repeat this $n - 1$ times, we conclude that P_n is true.

This principle is useful because it allows us to prove an infinite number of statements are true in just two easy steps! The statement P_N that we assume to hold is called the *induction hypothesis*. The key point in the induction step is to use the induction hypothesis, P_N, in order to deduce P_{N+1}.

Below are several examples to illustrate how to use this principle.

Example 1. Show that for $n = 1, 2, 3, \ldots$, the number $n^2 - n$ is even.

Solution. Base case. If $n = 1$, then $n^2 - n = 1^2 - 1 = 0$ and 0 is even.

Induction step. Our induction hypothesis is that $N^2 - N$ is even and we want to use this assumption to show that $(N+1)^2 - (N+1)$ is even. We write $(N+1)^2 - (N+1) = N^2 + 2N + 1 - N - 1 = N^2 - N + 2N$. Now $2N$ is even when N is an integer and $N^2 - N$ is even by our induction hypothesis. As the sum of two even numbers is again even, we conclude that $(N+1)^2 - (N+1)$ is even. □

Example 2. Show that for $n = 1, 2, \ldots$, we have

$$\sum_{j=1}^{n} 2j = n(n+1).$$

Solution. Base case. If $n = 1$, then the right-hand side of the claimed formula becomes $n(n+1) = 1 \cdot 2 = 2$. The left-hand side is

$$\sum_{j=1}^{1} 2j = 2.$$

Thus both sides are equal if $n = 1$, as claimed.

Induction step. Our induction hypothesis is that the formula $\sum_{j=1}^{N} 2j = N(N+1)$ is true. We have to show that $\sum_{j=1}^{N+1} 2j = (N+1)[(N+1)+1]$, which we rewrite more simply as

$$\sum_{j=1}^{N+1} 2j = (N+1)(N+2).$$

To prove this we write the last term in the sum on left-hand side separately and obtain

$$\sum_{j=1}^{N+1} 2j = \sum_{j=1}^{N} 2j + 2(N+1).$$

Now we use our induction hypothesis saying that $\sum_{j=1}^{N} 2j = N(N+1)$ to conclude that

$$\sum_{j=1}^{N+1} 2j = N(N+1) + 2(N+1).$$

Simplifying the right-hand side provides

$$N(N+1)+2(N+1) = N^2+N+2N+2 = N^2+3N+2 = (N+1)(N+2).$$

(Alternatively, we could factor out $(N+1)$ to obtain the same result:
$N(N+1)+2(N+1) = (N+2)(N+1)$.)
Thus, we have shown that the formula

$$\sum_{j=1}^{N+1} 2j = (N+1)(N+2).$$

is true. This completes the induction step and thus the proof by induction. □

Example 3. All horses are the same color.

Solution. We will show by induction that any set of N horses consists of horses of the same color.

The base case is easy. If we have a set with one horse, then all horses in the set are the same color.

We assume as our induction hypothesis that any set of N horses consists of horses of the same color. We take a set of $N+1$ horses, and put one of the horses in the barn for a moment. By our induction hypothesis, the remaining N horses are all of the same color. Now, we put a different horse in the barn. Again, the remaining N horses are all the same color. It follows that the set of $N+1$ horses are all the same color. □

Warning: This example is to illustrate that you should be careful in writing your solutions. Obviously, the above statement is wrong, so the above argument is **not correct**. Try to locate the error!

Worksheet 1 for MA 113 - Calculus I (Spring 08)

01/09/08

Work the following three problems related to mathematical induction and hand in your solutions. You will have time some time in recitation to begin working on these problems. Write up your solutions neatly, carefully and in complete sentences.

1. Find a formula that depends only on n to compute $\sum_{k=1}^{n}(2k-1)$ following the steps below:

(a) For $n = 1, 2, 3, 4$, compute

$$\sum_{k=1}^{n}(2k-1).$$

Make a guess for the value of this sum for $n = 1, 2, \ldots$.

(b) Use mathematical induction to prove that your guess is correct.

2. Use the principle of mathematical induction to prove that, for every positive integer n,

$$\sum_{k=1}^{n} 2k^2 = \frac{n(n+1)(2n+1)}{3}.$$

3. For $n = 1, 2, \ldots$, define the function f_n by $f_1(x) = x - 3$ and $f_{n+1}(x) = f_n(x) - 3$. (It is the principle of mathematical induction which tells us that these two statements suffice to define f_n for all n.) Use mathematical induction to prove that

$$f_n(x) = x - 3n.$$

Additional problems. Below are some additional exercises for you to consider. You will not be able to solve all of these problems at this time, so you are encouraged to come back to these problems later on.

All these additional problems will not be collected.

(1) Find the flaw in the proof that all horses are the same color.

(2) Let $f_1(x) = x - 2$ and then define f_n for $n = 1, 2, \ldots$ by $f_{n+1}(x) = f_1(f_n(x))$. (It is the principle of mathematical induction which tells us that these two statements suffice to define f_n for all $n = 1, 2, 3, \ldots$.) Use mathematical induction to prove that

$$f_n(x) = x - 2n.$$

(3) Show that if $r \neq 1$, we have

$$\sum_{k=0}^{n} r^k = \frac{1 - r^{n+1}}{1 - r}.$$

(4) Let P_n be the statement: $n^2 - n$ is an odd integer.

(a) Show that if P_n is true, then P_{n+1} is true.

(b) Is P_1 true?

(c) Is P_n true for any n?

(5) Let $f(x) = \sin(2x)$. Prove that for $n = 1, 2, \ldots,$

$$\frac{d^{2n}}{dx^{2n}} f(x) = (-4)^{2n} \sin(2x).$$

(6) Let $f(x) = xe^x$. Compute f', f'', and f'''. Guess a formula for the nth derivative,

$$\frac{d^n}{dx^n} f(x).$$

Prove that your guess is right.

(7) Argue that

$$\frac{d}{dx} x^n = nx^{n-1}, \qquad n = 1, 2, \ldots.$$

Hint: For the base case $n = 1$, use the definition of the derivative. For the induction step write $x^{n+1} = x \cdot x^n$ and use the product rule.

(8) Prove that

$$\frac{d}{dx} \frac{1}{x^n} = \frac{-n}{x^{n+1}}, \qquad n = 1, 2, \ldots.$$

(9) Prove that

$$\frac{d^n}{dx^n} x^n = n!, \qquad n = 0, 1, \ldots$$

(10) (a) Find a simple formula for

$$\sum_{k=1}^{n} ((k+1)^2 - k^2) = 2^2 - 1 + (3^2 - 2^2) + \cdots + n^2 - (n-1)^2 + (n+1)^2 - n^2.$$

(b) Using your answer to part (a), find a simple expression for

$$\sum_{k=1}^{n} (2k - 1).$$

To do this you should simplify each summand on the left.

(11) Use mathematical induction to prove that

$$\sum_{j=1}^{n} j^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

Duties of teaching assistants

1. In general, a recitation should consist of:

   (i) A period of reviewing the lectures and answering questions of the students. This should last no longer than 20 to 25 minutes.

   (ii) A period of group work devoted to solving problems. The goal is that the students learn to work on their own.

   The teaching assistant proposes the problems that are typically drawn from the common problems of the web homework sets or from the textbook. Students work in small groups on these problems. The teaching assistant will walk through the rows, give hints, and make suggestions to (groups of) students that are stuck. The teaching assistant may choose to ask a student to present the solution to a particularly interesting problem on the board.

   (iii) Practice quizzes as scheduled in the course schedule.

2. Make a brief syllabus which gives your name, office hours and contact information. Hand this out on the first meeting.

3. Grade pretests and return to the lecturer to be given to the student the next day. The scores on these tests may help to identify one or two students per section who have registered for calculus but are not prepared.

4. Beginning January 15 teaching assistants must take attendance in recitation in order to assign MA 193 grades. Accept any reasonable excuse for an excused absence. However, the student must make the effort to explain his or her absence.

   It might be useful to prepare a list of student names and take attendance by circulating this list to collect signatures.

5. Teaching assistants will be responsible for recording grades. We will record exam grades and written homework grades at mathclass.org. Be sure to download and/or print a copy of grades in the unlikely event that the servers melt down. Please provide a paper copy of the grades to the lecturer(s) that you are working with.

6. Prepare practice quizzes and solutions as pdf files. Using e-mail, distribute these files to each teaching assistant and lecturer by 3:30 pm the day before the quiz is to be given. A schedule will be prepared giving assignments for writing quizzes.

   Discuss a draft of the quiz and its solution with your corresponding lecturer before distributing the final version.

7. Teaching assistants should schedule one office hour in Mathskeller and two additional office hours at a location that is convenient for you and for your students.

8. Teaching assistants will help with proctoring and grading exams and will grade the written assignments.

9. Written assignments should be graded within one week. No written assignment should be handed in before an exam and returned after an exam.

10. Exams will be graded together the day after the exam starting at 8:30 am. Our goal is to complete our grading assignment by 5 pm.

11. Teaching assistants should provide a report to the coordinator on the scores for each exam. Please submit these reports by e-mail before 2 pm on the Thursday following a mid-term exam. For each section, this report should include four items:

    - The number of students who have taken the test.
    - The number of papers scoring above 30.
    - The mean score over all papers (including the ones scoring 30 or below).
    - The mean score of the papers scoring above 30.

    (Papers scoring 30 or below will be discarded in preparing a course average.)

    In addition, each lecturer may want additional information about her/his section.

12. If a student complains about a grade, ask that the student write down the question and hand it in with the paper. Bring the paper to your lecturer and to review the grading and make changes if necessary.

13. Answer feedback from within mathclass.org. Please try to answer messages once a day during the week. Our students should not expect responses on Saturday and Sunday.

    Please let the coordinator know if you run across a problem that appears to be incorrectly set.

14. The new mathclass.org seems to allow instructors to have different due dates for an assignment. Do *not* take advantage of this feature. If one section has a later due date, then students may join a section with an earlier due date, find the answers and submit these answers for credit.

    If there is a need to adjust a due date, please contact the coordinator and ask that the due date be changed for all sections. The homework assignments that are due the day before exams cannot be delayed.

15. Prepare grade-sheets and give to lecturer for signature.

16. Provide feedback to the coordinator about the progress of the students, ways of improving the course, or any other issues that come up.

17. At the end of the semester provide a table to the coordinator (and possibly to your lecturer) that lists for each student the score on the pretest and the final letter grade for the course.

Duties of lecturers

1. Make a brief syllabus which gives your name, office hours and contact information. Hand this out on the first day.

2. Deliver brilliant, insightful lectures exposing the beauty and power of Calculus.

3. Write exams.

4. Supervise invigilating and grading of exams. We will take turns as "exam tzar". A schedule will be distributed with exam writing and exam tzar assignments.

5. The exam tzar should: Prepare a key with point assignments for problems. Provide the coordinator with a clean copy of the key to scan and post on the web. Answer questions from graders. Review a few exams from each teaching assistant to make sure that the key is being followed and that following the key is producing sensible grades.

6. Each lecturer will handle appeals on grading in their sections. Report to the coordinator or exam czar if there appear to be serious, systematic mistakes in grading.

7. Review web homework problem sets, written assignments, and exam drafts and make suggestions for problems to be edited, deleted, or added.

8. Review practice quizzes written by your teaching assistants.

9. Participate in setting any curves on exams. Our goal is produce a course average by 3 pm on Thursday after each midterm exam. Information on the mean will be circulated by e-mail and each instructor will submit their suggestion for a curve. The curve will be the average of the suggested curves.

10. Keep a record of excused absences that caused a student to miss a homework assignment. I recommend that students who are excused from homework have their homework grade computed as an average of the assignments they submit.

11. Keep a record of the students' grades.

12. Work with teaching assistants to administer and grade makeup exams.

13. Visit recitations to help TA's develop as teachers. Follow up if serious problems are identified.

14. Review midterm and final grades and sign grade-sheets.

15. Circulate attendance sheets in lecture.

16. Provide feedback to the coordinator about the progress of the students, ways of improving the course, or any other issues that come up.

Duties of coordinator

1. Prepare common syllabus, written assignments, and their solutions.
2. Adjust due dates for homework, if needed.
3. Prepare schedules for exam tzars, writing exams, and writing practice quizzes.
4. Arrange for edits of web homework, if needed.
5. Prepare makeup exams.
6. Maintain common web site.

Suggestion for first recitation

The following is a suggested outline for our first recitation on Thursday, January 10.

- Welcome activity. (5 minutes) Collect names of students, other information if desired. Hand out teaching assistant contact information.
- Review selected material for review homework set, A1. (5 - 10 minutes) One should not expect to cover everything in the review set. Consider working one or more of the following examples.
  - Describing how to transform the graph of f to obtain the graph of $g(x) = f(ax + b)$.
  - Find the equation of a line. Be sure that students are familiar with the "point-slope" form for the equation of a line as well as the "slope-intercept" form.
  - Find a quadratic polynomial given the roots and one additional value. (Hint: Write the polynomial as $A(x - r_1)(x - r_2)$ where r_1 and r_2 are the roots.)
- Work in groups on homework or the above problems, answer questions as necessary. (15 - 20 minutes) (In theory, at the first lecture, the lecturer should have asked each student print out their assignment A1 and bring it to recitation.)

  It is very important to establish a tradition that time in recitation is to be spent working on mathematics. Students like to sit and be told what to do. However, to succeed on examinations, we need to work to develop our students' ability to find solutions independently.
- Administer pretest. (45 minutes) Allow students to leave when finished. This pretest must be graded, the scores recorded, and the papers given to the lecturer to return to students on Friday.

Solutions to pretest for Calculus I

Give one point per problem.
Please do not duplicate these answers and distribute them to students.

(1)	A	B	**C**	D	E
(2)	A	B	C	**D**	E
(3)	A	B	**C**	D	E
(4)	A	B	C	D	**E**
(5)	A	B	C	**D**	E
(6)	**A**	B	C	D	E
(7)	**A**	B	C	D	E
(8)	A	**B**	C	D	E
(9)	**A**	B	C	D	E
(10)	**A**	B	C	D	E
(11)	A	**B**	C	D	E
(12)	A	B	**C**	D	E
(13)	A	B	**C**	D	E
(14)	A	B	C	**D**	E
(15)	A	B	C	D	**E**
(16)	A	B	C	D	**E**
(17)	**A**	B	C	D	E

Pretest guidelines
01/07/08

All students will take a pretest in MA 113 this spring. It contains 17 questions. The test we use this semester is different from the previously used one that contained only 10 questions. We do not have data about the new test.

Below are the guidelines that were used to help students interpret their scores on the previously used pretest. It seems likely that, by scaling, the new pretest will lead to similar recommendations.

Students who score 8 or above in the pretest should be encouraged to continue in Calculus I.

Students who score below 5 should be discouraged to continue in Calculus I.

Students who score below 5 to 7.5 should plan on doing extra work to succeed in this course. This work might include

- working the warm-up assignments (AW, BW, CW, DW)
- routine exercises from the textbook
- joining the MathExcel section. Students interested in the MathExcel section should contact the coordinator or Peter Perry.

These recommendations should come from the lecturer, rather than a teaching assistant.

The table below (courtesy of Russell Brown) summarizes the performance in MA 113 of a number of students who have taken the pretest. Success is defined to mean earning A, B, or C. The information comes from 366 students in 19 sections over 4 semesters and 5 instructors. Note that this table only includes students who choose to stay in the course; it is likely that the probability of success is artificially high for students who obtain a low score on the pretest.

Score	Prob of succ	Fraction of class
10	89 %	10 %
9 - 9.5	80 %	17 %
8 - 8.5	77 %	16 %
7 - 7.5	80 %	17 %
6 - 6.5	49 %	11 %
5 - 5.5	50 %	13 %
4 - 4.5	32 %	8 %
3 - 3.5	55 %	5 %
0 - 2.5	7 %	4 %

A. $-\frac{6}{x}$ B. $-\frac{9}{8x}$ C. $\frac{9}{8x}$ D. $\frac{72}{x^6}$ E. $-\frac{6}{x^6}$

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9. What is the sum of the solutions of the equation $|x-4|=7$?

A. 8 B. 11 C. -3 D. 14 E. 0

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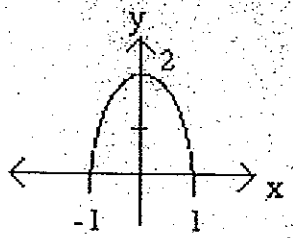
10. Which of the following is the equation of the parabola whose graph is shown below?

A. $y=-2x^2+2$ B. $y=-x^2+2$ C. $y=2x^2-2$ D. $y=x^2+2$
E. $y=-2x^2-2$



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11. If $f(x)=x^2+5$ and $g(x)=x-2$, then $(f\circ g)(x)$=?

A. $x^3-2x^2+5x-10$ B. x^2-4x+9 C. x^2+3
D. x^2+9 E. x^2+x+3

---

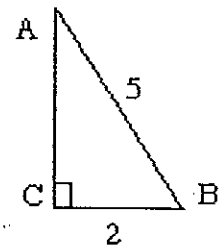
12. Which of the following is an equivalent form of $\sqrt[3]{8x^2}$?

A. $2x^2$ B. $2x^{3/2}$ C. $2x^{2/3}$ D. $8x^{3/2}$ E. $8x^{2/3}$

---

13. In right triangle below, $\overline{AB}$ has length 5 feet and $\overline{CB}$ has length 2 feet. What is the value of $\tan B$?

A. $\frac{\sqrt{21}}{5}$ B. $\frac{3}{2}$ C. $\frac{\sqrt{21}}{2}$ D. $\frac{2}{\sqrt{21}}$ E. $\frac{2}{5}$



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14. If $\sin\theta=\frac{3}{4}$, and $0<\theta<\frac{\pi}{2}$, then $\cos\theta$=?

A. $-\frac{6}{x}$ B. $-\frac{9}{8x}$ C. $\frac{9}{8x}$ D. $\frac{72}{x^6}$ E. $-\frac{6}{x^6}$

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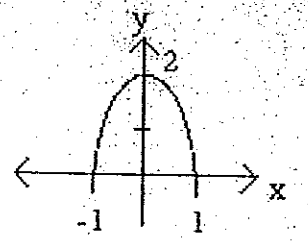

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13. In right triangle below, $\overline{AB}$ has length 5 feet and $\overline{CB}$ has length 2 feet. What is the value of $\tan B$?

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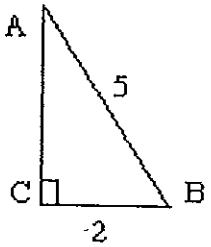

14. If $\sin\theta=\frac{3}{4}$, and $0<\theta<\frac{\pi}{2}$, then $\cos\theta$=?

Calculus Placement Test

1. What is the slope of the line with equation $3x+2y=7$?

A. $\frac{2}{3}$ **B.** $-\frac{2}{3}$ **C.** $\frac{3}{2}$ **D.** $-\frac{3}{2}$ **E.** $\frac{7}{2}$

---

2. What is the set of all values of x for which the expression $\frac{x+3}{x^2-4x}$ is not defined?

A. {-3} **B.** {0} **C.** {4} **D.** {0,4} **E.** {-3,0,4}

---

3. Which of the following is an equivalent form of $\frac{x}{x-2}-\frac{3}{x+2}$?

A. $\frac{x-3}{(x-2)(x+2)}$ **B.** $\frac{-3x}{(x-2)(x+2)}$ **C.** $\frac{x^2-x+6}{(x-2)(x+2)}$

D. $\frac{x^2-x-6}{(x-2)(x+2)}$ **E.** $\frac{x-3}{2x}$

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4. If $f(x)=x^2-2$, then $f(x+h)=?$

A. x^2+h^2 **B.** x^2-2+h **C.** x^2+h^2-2

D. $x^2+2xh+h^2$ **E.** $x^2+2xh+h^2-2$

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5. One of the factors of $15x^2-16x-7$ is

A. $5x+7$ **B.** $15x-1$ **C.** $5x+1$ **D.** $5x-7$ **E.** $3x-1$

---

6. The graphs of $x-2y=0$ and $3x+2y=24$ intersect in a point. What is the y-coordinate of that point?

A. 3 **B.** 6 **C.** 2 **D.** $\frac{3}{2}$ **E.** $\frac{5}{2}$

---

7. Which of the following sets is the domain of the function $f(x)=\sqrt{3x+4}$?

A. $\left[-\frac{4}{3},\infty\right)$ **B.** $\left(-\infty,-\frac{4}{3}\right]$ **C.** $[0,\infty)$ **D.** $\left[\frac{4}{3},\infty\right)$ **E.** $\left(-\infty,\frac{4}{3}\right]$

---

8. Which of the following expressions is a simplified form of $(-3x)^2(2x)^{-3}$?

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