Ma 111 all sections HW solutions

2.1

10. No name is given to the thing we call.

16. First we have the name "crook", second its description in known words, and third the bi-conditional relation since a "person who steals or cheats" is a "crook".

22. This remark is not a statement since whether or not a joke is "great" is a matter of taste and not of logic. To be a statement we should say to whom the jokes are great.

26. This demand is not a statement since following a demand is a matter of choice and not of logic. To be a statement it should say that the formula is the only way to solve the problem.

30. The two lines are not parallel.

32. There are rational numbers that are irrational.

42. The converse: if it becomes flat, there will be a leak in the tube. The inverse: if there is no leak in the tube, it will not become flat. The contrapositive: if it does not become flat, there will not be a leak in the tube.

50. The answer may vary. To give an example, "If I answer this question, the question is answered by me" is a true statement whose inverse "If do not answer the question, the question is not answered by me" is also true.

2.2

10. All N are T and R is T does not imply that R is N. A good change is all N are T and R is not T so that R is not N. For example, we could say "Russell is 4.9 feet tall" so "Russell is not an NBA player".

12. This is correct since all R are B and M is R implies M is B.

16. All people who steal are dishonest and shoplifters are stealing so that shoplifters are dishonest.

18. If $3(x+4)=18$, then $x=2$.

20. If A, then $\sim E$.

24. The experiment should conclude that perhaps the line joining the midpoints of two sides is about half the third side. The measurements may vary.

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2.3

8. $N =$ NBA player, $A =$ over 5 feet tall,

$$\begin{array}{c} N \to A \\ N \\ \hline \therefore A \end{array}$$

10. $S =$ square, $F =$ four sides,

$$\begin{array}{c} S \to F \\ \sim F \\ \hline \therefore \sim S \end{array}$$

18.

A	C	$\sim C$	$C \vee \sim C$	$A \to (C \vee \sim C)$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	T
F	F	T	T	T

22.

A	B	$B \to A$	$\sim A$	$\sim B$	$\sim A \to \sim B$
T	T	T	F	F	T
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	T	T

The statements are equivalent.

26.

P	Q	$\sim Q$	$\sim P$	$\sim Q \to \sim P$	$(\sim Q \to \sim P) \wedge P$	$[(\sim Q \to \sim P) \wedge P] \to Q$
T	T	F	F	T	T	T
T	F	T	F	F	F	T
F	T	F	T	T	F	T
F	F	T	T	T	F	T

The argument is correct.

28.

B	C	$\sim C$	$B \to \sim C$	$B \wedge (B \to \sim C)$	$[B \wedge (B \to \sim C)] \to \sim C$
T	T	F	F	F	T
T	F	T	T	T	T
F	T	F	T	F	T
F	F	T	T	F	T

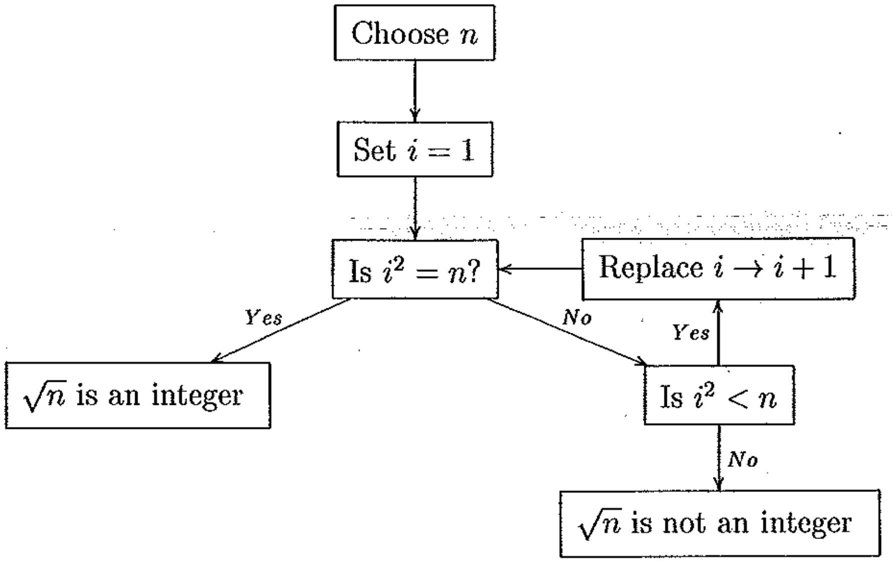
The argument is correct.

2.4

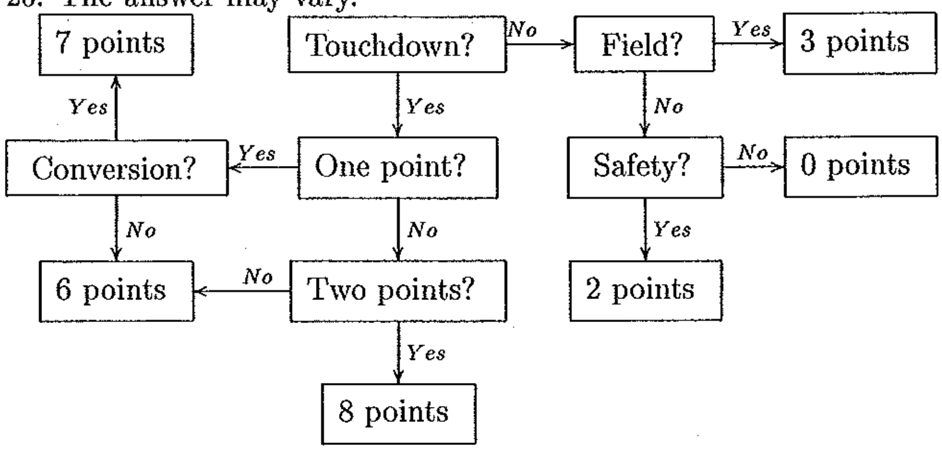
8. This flowchart describes the process which helps decide why a TV will not turn on. The choices are: the TV is not plugged in, the receptacle is not working, the TV is not working.

10. a) It will be placed in desk. b) It will be inserted in sharpener and then placed in desk. c) It will be placed in desk or discarded. d) It will be inserted in sharpener and then placed in desk or placed in desk directly. e) It will be discarded after being inserted repeatedly in sharpener.

16. The answer may vary.



20. The answer may vary.



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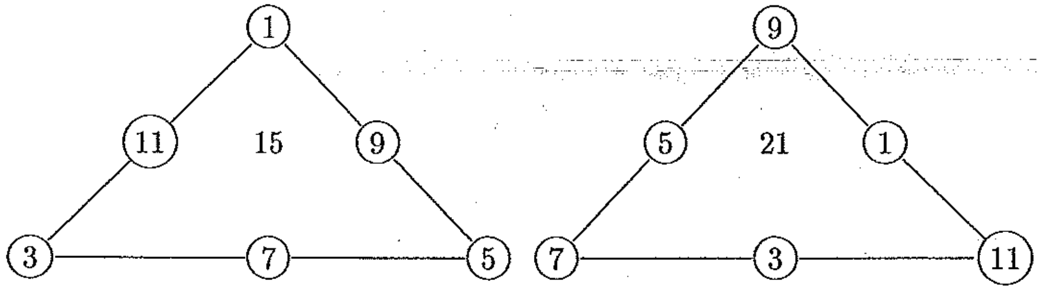
2.5

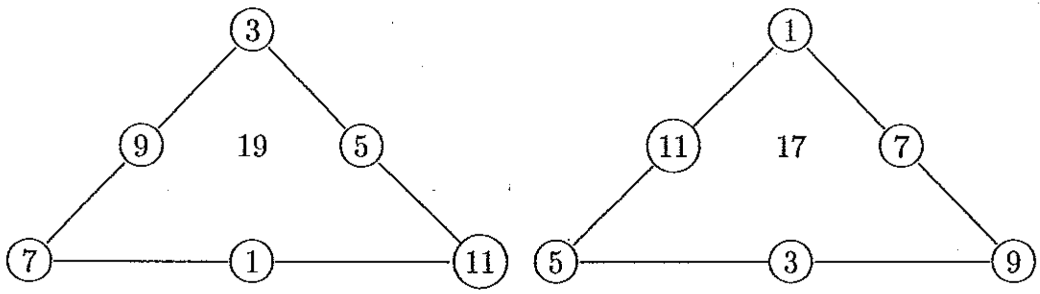
10. A possible "big-small" pattern could be:

(BS)



A possible say "one-three" pattern could be:

(OT)



where a number a is of type "one" if $a-1$ is divisible by 4 and "three" otherwise.

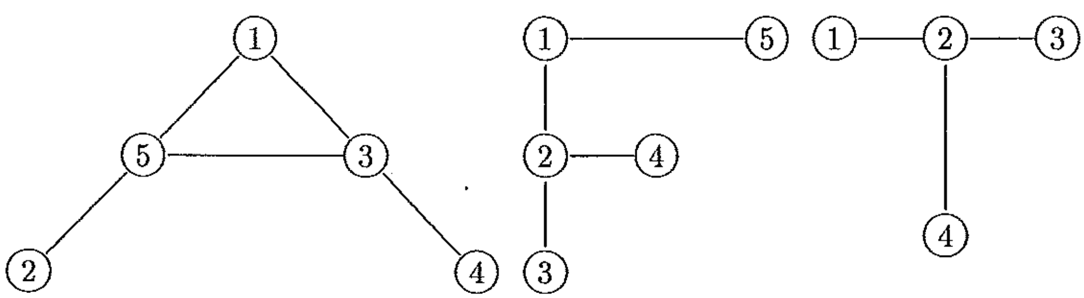
12. The sum should be a third of $2+4+6+8+10+12+14+16+18$, i. e. 30. Then we decompose 30 into a sum and look for how often terms occur in various such decompositions:

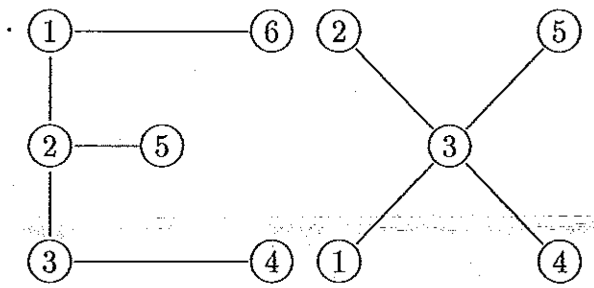
$$2+10+18$$
$$2+12+16$$
$$4+10+16$$
$$4+12+14$$
$$4+18+8$$
$$6+10+14$$
$$6+16+8$$
$$8+10+12$$

Since 10 occurs four times it should be in the center. Also 2 and 6 should not be in a corner.

16	6	8
2	10	18
12	14	4

18. By "balancing" big and small numbers, we get





22. By carrying, say $T = 1$ and A and E are small like 2 and 3. Then let $I = 9$ and find

$$\begin{array}{cccc} & 8 & 9 & 5 \\ + & 4 & 3 & 2 \\ \hline 1 & 3 & 2 & 7 \end{array} \quad \text{or} \quad \begin{array}{cccc} & 7 & 9 & 4 \\ + & 5 & 3 & 2 \\ \hline 1 & 3 & 2 & 6 \end{array}$$

24. Say $T = 0$ and observe that A and N are like 4 and 6. Hence,

$$\begin{array}{cccc} & 1 & 4 & 0 \\ \times & & & 6 \\ \hline & 8 & 4 & 0 \end{array}$$

30. We need to divide 24 and get a number smaller than 7. Say $24 \div 4 = 6$ or $24 \div 6 = 4$. Then try $2 \times 6 - 7 = 5$ and $5 \times 1 + 2 = 7$.

5	$\times$	1	+	2	7
$\times$		+		$\times$	
3	+	24	$\div$	6	7
$-$		$\div$		$-$	
8	+	4	$-$	7	5
7		7		5	

3.1

12. a) $\{x |\ x \text{ is an odd whole number with } x \leq 12\}$

b) $\{1, 3, 5, 7, 9, 11\}$

14. Correct symbol but not true. $F \subset I$ is true.

16. True and correct.

18. True but incorrect symbol. $5 \in F$ is correct.

20. Correct symbol but not true. $5 \in F$ is true.

26. Assign to each number $n = 4k - 3$ the natural number $k = (n + 3)/4$. In this way we get a one-to-one correspondence between the two sets of numbers.

30. $E = \{$*Belgium*, *Luxemburg*, *Germany*, *Switzerland*, *Italy*, *Monaco*, *Andorra*, *Spain*$\}$.

Solutions to Sec 4.1 and Sec 4.2

Sec 4.1

(12) (a) There is only one 7 of spades, so probability $= \frac{1}{52}$

(b) There are four 7's in a deck, so probability $= \frac{4}{52} = \frac{1}{13}$

(c) There are twelve face cards in a deck(three in each suit), so probability $= \frac{12}{52} = \frac{3}{13}$

(d) There are thirteen hearts in a deck, so probability $= \frac{13}{52} = \frac{1}{4}$

(e) There are twenty six red cards (thirteen each for hearts and diamonds), so probability $= \frac{26}{52} = \frac{1}{2}$

(f) There are four 8's and four aces, so probability $= \frac{8}{52} = \frac{2}{13}$

(14) (a) There are five ways in which you can have a sum 8 with two dice(2+6, 3+5, 4+4, 5+3, 6+2) and there are 36 different possibilities with two dice, so probability $= \frac{5}{36}$

(b) There is no combination of two dice which has sum 1, so probability $= \frac{0}{36} = 0$

(c) There are ten ways in which this is possible ((1, 3), (2, 3), (4, 3), (5, 3), (6, 3), (3,1), (3, 2), (3, 4), (3, 5), (3, 6)), so probability $= \frac{10}{36} = \frac{5}{18}$

(d) There is no way that the sum of two dice is 13(The maximum is $6+6 = 12$), so probability $= \frac{0}{36} = 0$

(e) Any sum of two dice is always less than 13 so probability $= 1$

(f) From part (a) the chance of having a sum of 8 in a roll is $\frac{5}{36}$ therefore if it is rolled 9000 times then the chance of having a sum of 8 is $9000 \times \frac{5}{36} = 1250$

(16) The probability of reaching base is 0.435. There if he comes to bat 620 times, then the number of times he is expected to reach base is $620 \times 0.435 = 270$

(18) There is 75% of waking up when the alarm rings, therefore the chance of not waking up when the alarm rings is given by $100\% - 75\% = 25\%$

(22) (a) There are 12 possibities from each die. Therefore the possible

outcomes with two dice is $12 \times 12 = 144$

(b) There is only one way that the sum of the two dice can be 24 (12+12), therefore the probability $= \frac{1}{144}$

(c) There is no combination of two dice which has sum more than 24 , therefore the probability $= 0$

(d) There are eight combinations of two dice having a sum 17((5, 12), (6, 11), (7, 10), (8, 9), (9, 8), (10, 7), (11, 6), (12, 5)), therefore probability $= \frac{8}{144} = \frac{1}{18}$

(e) Combination which has sum 18 = (6, 12), (7, 11), (8, 10), (9, 9), (10, 8), (11, 7), (12, 6)

Combination which has sum 19 = (7, 12), (8, 11), (9, 10), (10, 9), (11, 8), (12, 7)

Combination which has sum 20 = (8, 12), (9, 11), (10, 10), (11, 9), (12, 8)

Combination which has sum 21 = (9, 12), (10, 11), (11, 10), (12, 9)

Combination which has sum 22 = (10, 12), (11, 11), (12, 10)

Combination which has sum 23 = (11, 12), (12, 11)

Combination which has sum 24 = (12, 12)

Therefore total number of ways of having a sum greater than 17 = 28

Hence probability $= \frac{28}{144} = \frac{7}{36}$

(f) Any combination of two dice always has sum greater than 1, therefore probability $= 1$

(24) (a) There are eight kings, therefore probability to draw a king on the first card $= \frac{8}{48} = \frac{1}{6}$

(b) Now there are 7 kings left and there are 47 total cards left, therefore probability of drawing another king $= \frac{7}{47}$

(c) Now there are 6 kings left and there are 46 total cards left, therefore probability of drawing another king $= \frac{6}{46} = \frac{3}{23}$

Sec 4.2

6 (a) Number of ways of dealing 3 Kings $= {}^{4}C_{3} = \frac{4!}{3!\,1!} = 4.$

Number of ways of dealing 2 other cards $= {}^{48}C_{2}$

$= \frac{48!}{2!\,46!}$

$= \frac{48 \times 47}{2 \times 1} = 1128$

Number of ways of dealing any 5 cards $= {}^{52}C_{5}$

$= \frac{52!}{5!\,47!}$

$= 2598960$

$\therefore$ Probability of dealing 3 kings $= \frac{4 \times 1128}{2598960} = \boxed{\frac{1}{576}}$

(b) From part (a) probability of dealing 3 kings $= \frac{1}{576}$

There are 13 ranks, therefore probability of dealing three of any rank $= \boxed{\frac{13}{576}}$

8 (a) Number of ways of dealing an ace $= {}^{8}C_{1} = \frac{8!}{7!\,1!} = 8$

Number of ways of dealing four kings $= {}^{8}C_{4} = \frac{8!}{4!\,4!} = 70$

Number of ways of dealing 5 cards $= {}^{104}C_{5} = \frac{104!}{5!\,99!}$

$= 91962520$

$\therefore$ Probability of dealing an ace and 4 kings $= \dfrac{8 \times 70}{91962520}$

$= \boxed{\dfrac{1}{164,219}}$

(b) Number of ways of dealing an ace $= {}^8C_1 = \dfrac{8!}{1!\,7!} = 8$

From part (a) Number of ways of drawing 4 kings $= {}^8C_4$

$= 70$

But there are 12 other ranks, therefore number of ways of drawing 4 cards of same rank $= 12 \times 70$

$\therefore$ Probability of dealing an ace and 4 cards of same rank

$= \dfrac{8 \times 12 \times 70}{91962520}$

$= \boxed{\dfrac{1}{13,685}}$

10(a) Number of ways of drawing 3 kings $= {}^4C_3 = \dfrac{4!}{3!\,1!} = 4$.

Number of ways of drawing 2 queens $= {}^4C_2 = \dfrac{4!}{2!\,2!} = 6$

Number of ways of drawing 5 cards $= {}^{52}C_5 = \dfrac{52!}{5!\,47!} = 259896$

$\therefore$ Probability of drawing a full house with 3 kings and 2 queens $= \dfrac{4 \times 6}{2598960}$

$= \boxed{\dfrac{1}{108,290}}$

(b) Number of ways of dealing 3 kings $= {}^4C_3 = 4$.

Number of ways of dealing two cards of another rank
$= 12 \times {}^4C_2 = 12 \times 6 = 72$

$\therefore$ Probability $= \dfrac{4 \times 72}{2598960} = \boxed{\dfrac{1}{9024}}$

(c) Number of ways of dealing 3 cards of any rank $= 13 \times {}^4C_3 = 52$
Number of ways of dealing 2 cards of any rank $= 12 \times {}^4C_2 = 72$

$\therefore$ Probability $= \dfrac{52 \times 72}{2598960} = \boxed{\dfrac{1}{694}}$

(12) There is only 1 trifecta.

Number of ways of selecting the first three horses $= {}^6P_3 = \dfrac{6!}{3!} = 120$

$\therefore$ Probability of picking a trifecta $= \boxed{\dfrac{1}{120}}$

(14)(a) Number of #'s which are not winning #'s = 60
Number of ways of selecting 10 non-winning #'s $= {}^{60}C_{10}$

Number of ways of selecting any 10 #'s $= {}^{80}C_{10}$
$\therefore$ Probability that there is no correct number selected

$= \dfrac{{}^{60}C_{10}}{{}^{80}C_{10}}$

$= \boxed{0.04579}$

(6) Number of ways of selecting 6 correct #'s = $^{20}C_6$

Number of ways of selecting 14 non-winning #'s

$= {}^{60}C_{14}$.

$\therefore$ Probability $= \dfrac{^{20}C_6 \times {}^{60}C_{14}}{^{80}C_{20}} = \boxed{0.01148}$

(18) Number of ways of selecting any two cities $= {}^{18}C_2$

$= \dfrac{18 \times 17}{2 \times 1}$

$= 153$.

There is only 1 combination of two westernmost cities

$\therefore$ Probability $= \boxed{\dfrac{1}{153}}$

(20) There is only 1 ways to select two oldest members.

Number of ways of selecting 2 members from 8 people

$= {}^8C_2 = \dfrac{8 \times 7}{2 \times 1} = 28$

$\therefore$ Probability $= \dfrac{1}{28}$

(22) There is only 1 way to hang the paintings in alphabetical order

Now number of ways to hang 14 paintings $= {}^{14}P_{14} = \dfrac{14!}{0!} = 14!$

$\therefore$ Probability $= \dfrac{1}{14!} = \dfrac{1}{87,178,291,200}$

(24) There is only one way in which all the 10 people get off at the last stop

But every person has 3 choices to get off the bus.

$\therefore$ 10 people have 3^{10} choices to get off the bus.

$\therefore$ Probability $= \dfrac{1}{3^{10}}$

Sec 4.3

8 Odds = 1 to 3.

Probability of the event $= \frac{1}{1+3} = \frac{1}{4}$

10 Probability $= \frac{1}{10} = P(E)$

$P(\bar{E}) = 1 - \frac{1}{10} = \frac{9}{10}$

$\therefore$ Odds $= \frac{P(E)}{P(\bar{E})} = \frac{\frac{1}{10}}{\frac{9}{10}} = \frac{1}{9}$ (1 to 9).

12 House odds = 20 to 1

Probability that house wins $= \frac{20}{20+1} = \frac{20}{21}$

Probability that you win $= \frac{1}{20+1} = \frac{1}{21}$

14 (a) 75% chance of getting up $= \frac{3}{4}$

$\therefore$ Probability that you will not get up = 100% − 75% = 25% $= \frac{1}{4}$.

(b) odds of not getting up $= \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} = 1$ to 3.

(c) odds of getting up $= \frac{\frac{3}{4}}{\frac{1}{4}} = \frac{3}{1} = 3$ to 1

16 (a) Probability of a gene having a defect $= \frac{1}{25}$

Probability of the gene of not having a defect $= 1 - \frac{1}{25} = \frac{24}{25}$

$\therefore$ Odds of the gene having that defect $= \frac{\frac{1}{25}}{\frac{24}{25}} = \frac{1}{24} = 1$ to 24

(b) Probability of the gene not having that defect $= \frac{24}{25}$.

18(a) Probability of Harward winning the Ivy League Champ. $= \frac{1}{25}$
Probability of Harward not winning the champ. $= 1 - \frac{1}{25} = \frac{24}{25}$.

odds of winning the championship $= \dfrac{\frac{1}{25}}{\frac{24}{25}} = \frac{1}{24} = 1$ to 24

(b) Probability of Harward not winning the champ. $= \frac{24}{25}$

(c) Odds of not winning the championship $= \dfrac{\frac{24}{25}}{\frac{1}{25}} = \frac{24}{1} = 24$ to 1

(d) The casino would post the odds of Harward not winning the championship so it would be 24 to 1

Sec 4.4

8 $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.78 + 0.29 - 0.21 = 0.86$

10 $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.56 + 0.48 - 0.86 = 0.18$

12 This is ~~because~~ not possible because $P(A \cap B)$ cannot be greater than $P(B)$

14 This is not possible because $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.58 + 0.32 - 0.92$
$= -0.02$ which is negative
hence not possible.

16 Probability that you fly to Mexico $= \frac{8}{20} = \frac{2}{5}$

20 Probability that you take a cruise or go to Hawaii
$P(C \cup H) = P(C) + P(H) - P(C \cap H)$
$= \frac{5}{20} + \frac{9}{20} - \frac{2}{20} = \frac{12}{20} = \frac{3}{5}$

22 Probability that you fly or go to Mexico

$= P(F \cup M) = P(F) + P(M) - P(F \cap M)$

$= \frac{15}{20} + \frac{11}{20} - \frac{8}{20} = \frac{18}{20} = \frac{9}{10}$

28 Probability of Red sox winning series in four games

$= (0.4)(0.4)(0.4)(0.6) \times 3$

$= 0.1152$

30 Probability of Red sox winning the series

$= P(\text{win in 3 games}) + P(\text{Win in 4 games}) + P(\text{Win in 5 games})$

$P(\text{win in 4 games}) = 0.1152$

$P(\text{Win in 5 games}) = 6 \times (0.4)(0.4)(0.4)(0.6)(0.6)$

$= 0.13824$

$P(\text{win in 3 games}) = (0.4)(0.4)(0.4)$

$= 0.064$

$\therefore$ Probability of Red sox winning the series $= 0.1152 + 0.13824 + 0.064$

$= 0.31744$

32 Probability of the Yankees not winning the series in 4 games

$= 1 -$ Probability that Yankees win series in 4 games

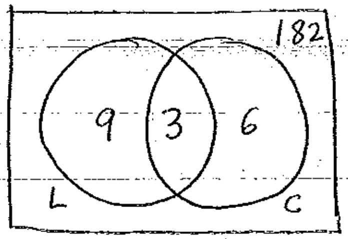
$= 1 - 0.2592$ (See Ex 7(b))

$= 0.7408$

34 Probability that the series ends in 4 games = P(Red sox win series in 4 games) + P(Yankees win series in 4 games)

$= 0.1152 + 0.2592 = 0.3744$

38 (a)



L = Leukemia
C = Cataracts

(b) Probability that a cat had leukemia but not Cataracts

$= \frac{9}{200}$

(c) Prob. that a cat had cataracts ~~but~~ but not leukemia

$= \frac{6}{200}$

(d) Prob. that a cat ~~had~~ had neither cataract nor leukemia

$= \frac{182}{200}$.

Sec 4.5

8. $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.21}{0.63} = \frac{1}{3}$

10. $P(A|B) = \frac{P(A\cap B)}{P(B)}$

$\Rightarrow P(A\cap B) = P(A|B) \cdot P(B)$

$= 0.21 \times 0.29$

$= 0.0609$

12. $P(A\cap B) = P(A) + P(B) - P(A\cup B)$

$= 0.78 + 0.29 - 0.86 = 0.21$

$P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{0.21}{0.78} = 0.2692$

16. No. of ways of winning a trip to Mexico = 11

" " " " " plane trip to Mexico = 8

$\therefore$ Probability $= \frac{8}{11}$

18. Total no. of males = 1200

Total no. of males in ~~Proff~~ Professional Speciality = 163.

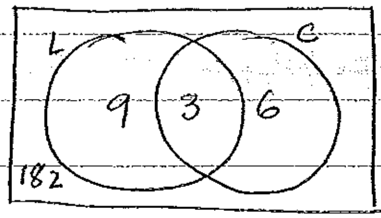
$\therefore$ Probability $= \frac{163}{1200}$

20. Total no. of people in Professional speciality = 397

No. of females in " " = 234.

$\therefore$ Probability $= \frac{234}{397}$

24 (a)

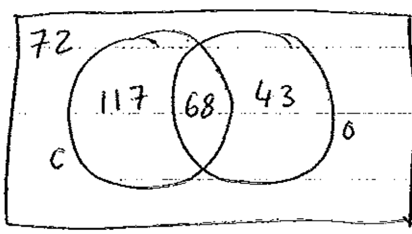


L = Leukemia
C = Cataracts.

(b) No. of cats having cataracts = 9.
No. of cats having both cataracts and leukemia = 3.
$\therefore$ Probability $= \frac{3}{9} = \frac{1}{3}$

(c) No. of cats having leukemia = 12
No. of cats having both leukemia and cataracts = 3
$\therefore$ Probability $= \frac{3}{12} = \frac{1}{4}$.

26 (a)



C = Coffee
O = Orange juice.

(b) No. of people drinking coffee = 117 + 68 = 185
No. of people drinking both = 68.
$\therefore$ Probability $= \frac{68}{185}$

(c) No. of people not drinking orange juice = 72 + 117 = 189
No. of people drinking only coffee = 117.

$\therefore$ Probability $= \frac{117}{189}$

Sec 4.6

8 $\quad E.V. = 0.3 \times 50 + 0.7 \times (-5)$

$= 15 - 3.50$

$= \$11.50$

Expected winnings after playing 100 times $= 100 \times 11.50$

$= \$1150$

10 Player loses $= x$.

$$0 = 8 \times \frac{1}{36} + 2 \times \frac{9}{36} + x \times \frac{26}{36}.$$

$$\Rightarrow -\frac{26x}{36} = \frac{26}{36}$$

$$\Rightarrow x = -1$$

$\therefore$ The player should lose \$1.

12 (a) $E.V = \sum$ Probability $\times$ Amount won [sum up for all 6 cases]

$= -0.53722$

(b) After 1000 games the player should expect to lose

$1000 \times .53722 = \$537.22$

14 Let value of third prize $= x$.

$$\therefore E.V. = 1 = (2 - 0.50) \times 0.35 + (2-1) \times 0.15 + (2-x) \times 0.01 + 2 \times 0.49$$

$$\Rightarrow 1 = 1.50 \times 0.35 + 1 \times 0.15 + 0.02 - .01x + 0.98$$

$$\Rightarrow \cancel{1} = .525 + .15 - .01x + \cancel{1}$$

$$\Rightarrow .01x = .675$$

$$\Rightarrow x = \$67.50$$

16 Life Expectancy $= 1\times 0{\cdot}14 + 2\times 0{\cdot}07 + 3\times 0{\cdot}26 + 4\times 0{\cdot}29 + 5\times 0{\cdot}24$
$= 3{\cdot}42$ yrs.

18 $EV = \frac{1}{38}\times 35 + \frac{37}{38}\times(-1)$
$= -\frac{2}{38} = -\frac{1}{19} = -\$0.0526$. [Player's loss]

No. of times they play in total $= 24\times 10\times 100 = 24{,}000$
$\therefore$ Casino's profit $= 24000 \times \cdot 0{\cdot}0526$
$= \$\,1262{\cdot}40$

20 The answer is $102·14.

22 Let probability of winning in the next round $= x$
$\therefore E.V. = 0 = x\times(50{,}000 - 12{,}500) + (1-x)(-12{,}500)$
$\Rightarrow\ 0 = 37{,}500x - 12{,}500 + 12{,}500x$.
$\Rightarrow\ 12{,}500 = 50{,}000x$.
$\Rightarrow\ x = \frac{12{,}500}{50{,}000} = \frac{1}{4} = 0{\cdot}25$.

§3.2 Probs 10, 12, 18, 20, 22, 26 Solns

10. $F = \{$oranges, apples, apricots, peaches$\}$

$N = \{$coconuts, filberts, almonds$\}$

a. $F \cup N = \{$oranges, apples, apricots, peaches, coconuts, filberts, almonds$\}$

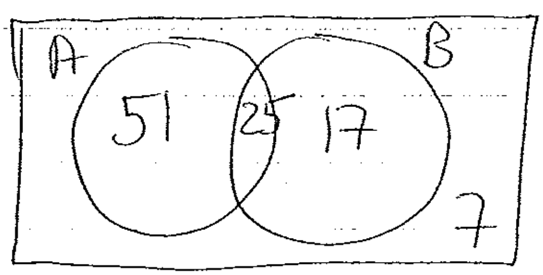
b. $F \cap N = \emptyset$ The sets have nothing in common.

12. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 32 + 38 - 7 = 63$

So $n(A \cup B) = 63$.

18.

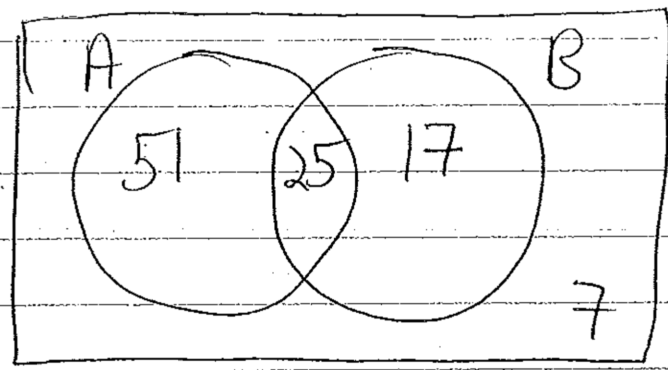


$n(B) = 25 + 17 = 42$

20. $n(A \cap B) = 25$

(The overlap of the circles represents $A \cap B$.)

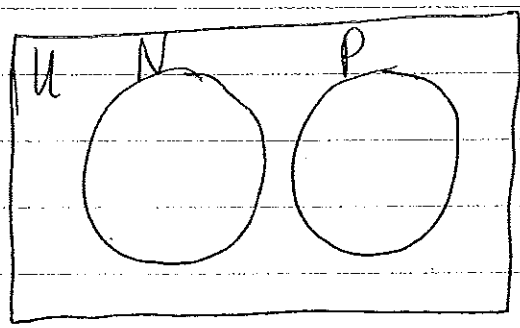
22.



$\overline{A}$ is the region outside A, which are the regions with the 17 and 7 marked.

Thus, $\overline{A} \cap B$ is the region outside A + inside B. So $n(\overline{A} \cap B) = 17$.

26.



The circles don't overlap since there are no books that are both novels + books of poetry.

In N, we have novels.
In P, we have books of poetry.
In U but outside N + P are the other kinds of books in the library (encyclopedias, for example).

Section 9.1

(10) (a) $9\% = 0.09$ (b) $0.7\% = 0.007$ (c) $17.5\% = 0.175$

(d) $92.16\% = 0.9216$ (e) $120\% = 1.20$

(12) (a) $0.02 = 2\%$ (b) $0.123 = 12.3\%$ (c) $0.6075 = 60.75\%$

(d) $1.34 = 134\%$ (e) $0.003 = 0.3\%$

(14) (a) $3/4 = 0.75 = 75\%$ (b) $7/8 = 0.875 = 87.5\%$ (c) $31/50 = 0.62 = 62\%$

(d) $21/32 = 0.65625 = 65.625\%$ (e) $39/40 = 0.975 = 97.5\%$

(18) (a) $100 - 20 = 80$

$$\frac{80}{100} = \frac{74}{x}$$

$80x = 7400$

$x = \$92.50$

(b) $100 - 30 = 70$

$$\frac{70}{100} = \frac{74}{x}$$

$70x = 7400$

$x = \$105.71$

(c) $100 - 45 = 55$

$$\frac{55}{100} = \frac{74}{x}$$

$55x = 7400$

$x = \$134.55$

(20) $100 + 60 = 160$

(a)
$$\frac{160}{100} = \frac{x}{1.79}$$

$100x = 286.4$

$x = \$2.86$

(b)
$$\frac{160}{100} = \frac{x}{2.99}$$

$100x = 478.4$

$x = \$4.78$

(c)
$$\frac{160}{100} = \frac{x}{3.50}$$

$100x = 560$

$x = \$5.60$

(22) $100 + 40 = 140$

(a)
$$\frac{140}{100} = \frac{79.95}{x}$$

$140x = 7995$

$x = \$57.11$

(b)
$$\frac{140}{100} = \frac{285.99}{x}$$

$140x = 28599$

$x = \$204.28$

(c)
$$\frac{140}{100} = \frac{1599}{x}$$

$140x = 159900$

$x = \$1142.14$

(26)
$$\frac{102.3}{100} = \frac{x}{175}$$

$100x = 17902.5$

$x = \$179.03$

(28)
$$\frac{135}{100} = \frac{x}{100}$$

$100x = 13500$

$x = 135$

$$\frac{65}{100} = \frac{x}{135}$$

$100x = 8775$

$x = \$87.75$

$100 - 87.75 = \$12.25$

(30) $70{,}000 - 65{,}550 = 4{,}450$

$$\frac{30.5}{100} = \frac{x}{4450}$$

$100x = 135725$

$x = \$1357.25$

$14{,}645 + 1357.25 = \$16{,}002.25$

(32) (a)
$$\frac{2.75}{54.50} = \frac{x}{100}$$

$54.5x = 275$

5.0467

(b)
$$\frac{1.5}{20.75} = \frac{x}{100}$$

$20.75x = 150$

$x = 7.2292$

Section 9.2

(8) $I = 3000(.04)(6) = \$720$

(10) $I = 3500(.005)(6) = \$105$

(12) $A = 2000(1 + .05(4)) = \$2400$

(14) $A = 25{,}000(1 + .05(.25)) = \$25{,}312.50$

(18) $6000 = P(1 + .05(4))$

$P = \frac{6000}{1.2} = \$5000$

(20) $2350 = 2000(1 + .05t)$

$1.175 = 1 + .05t$

$.175 = .05t$

$t = 3.5$ years

(22) $2600 = 2500(1 + 3r)$

$1.04 = 1 + 3r$

$.04 = 3r$

$r = .01\overline{3} = 1.33\%$

(24) $A = 4700(1 + .096(.5)) = \$4808.10$

(26) $26.37 = 23.63(1 + 1.5r)$

$1.11595 = 1 + 1.5r$

$.11595 = 1.5r$

$r = .0773 = 7.73\%$

(28) $7000 = 6350(1 + r)$

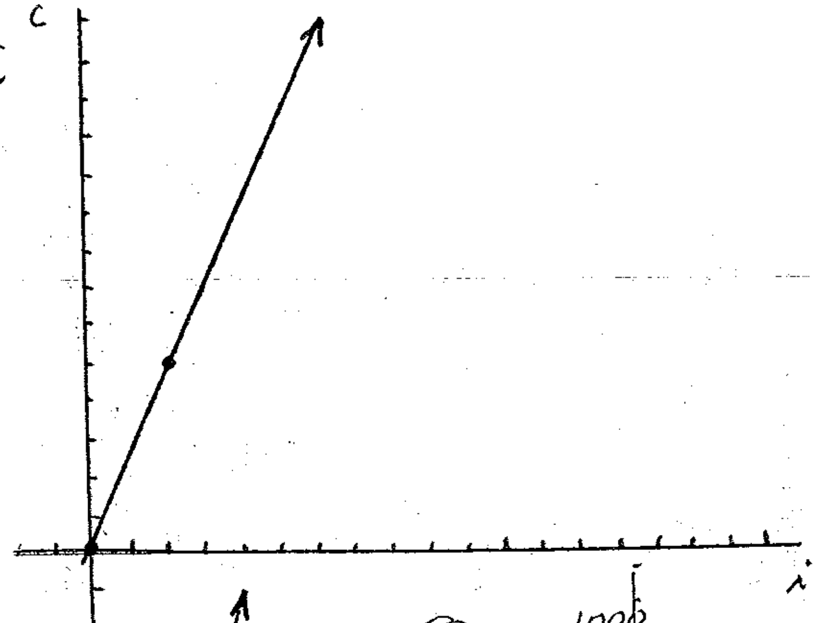
$1.10236 = 1 + r$

$r = .10236 = 10.24\%$

Section 6.1

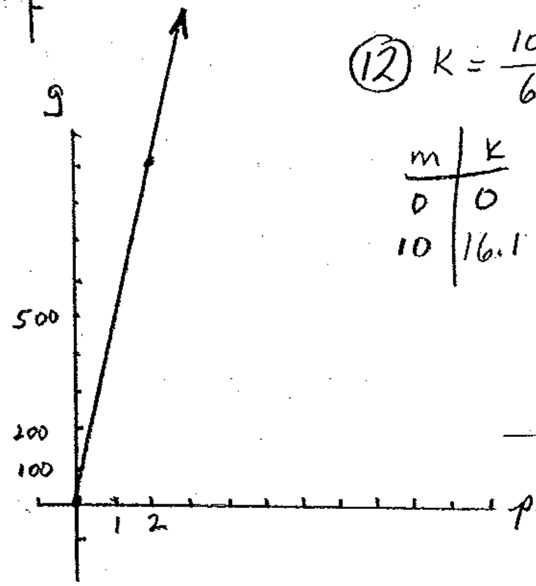
(8) $C = 2.54i$

i	c
0	0
2	5.08



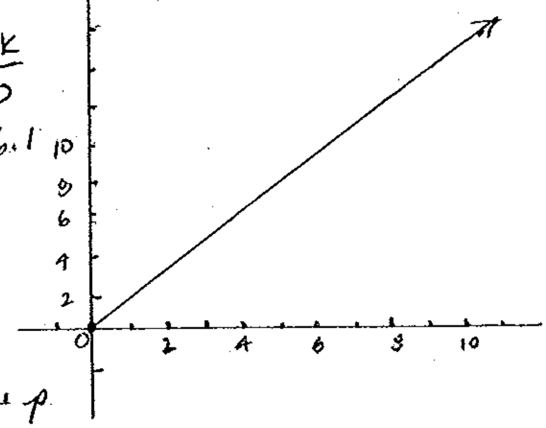
(10) $g = 454p$

p	g
0	0
2	908



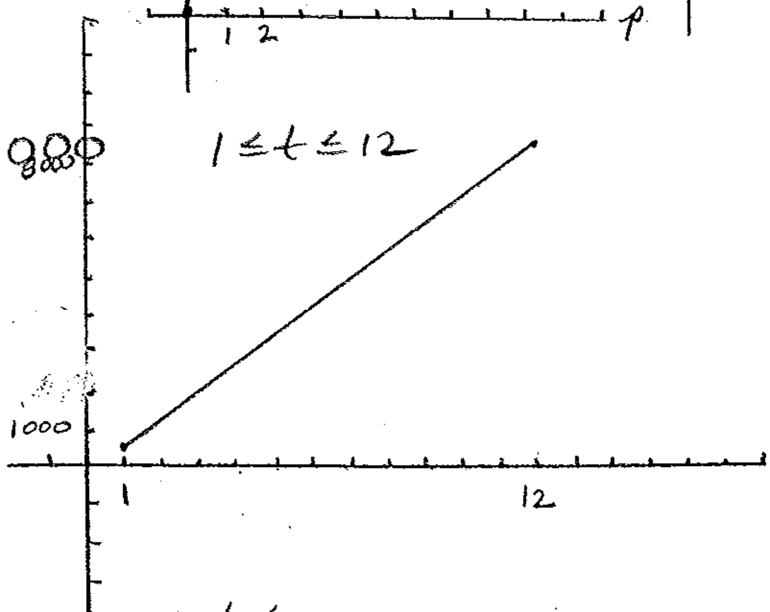
(12) $K = \frac{1000}{621} m$

m	K
0	0
10	16.1



(14) $P = 750t - 1000 \quad 1 \le t \le 12$

t	P
1	-250
12	8000



(16) $P = -750t - 1000 \quad 1 \le t \le 12$

t	P
1	-1750
12	-10,000

(18) $4x - 3y = 4$

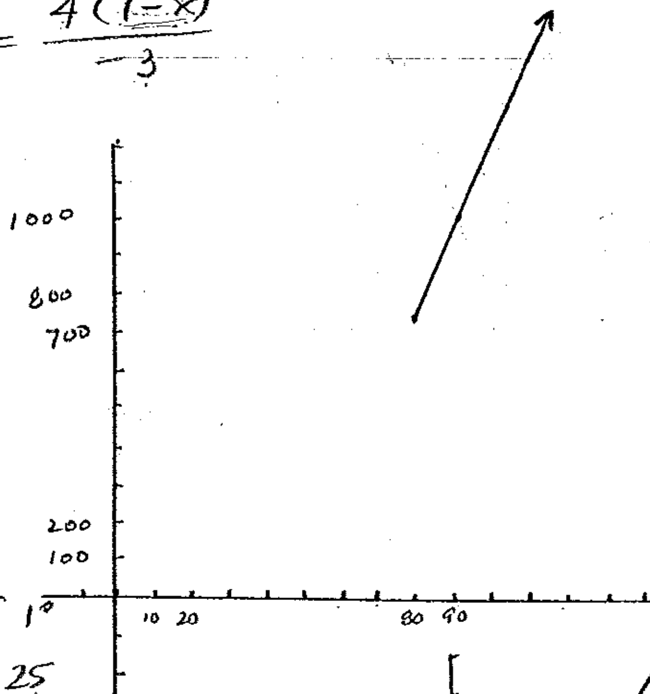
$-3y = 4 - 4x$

$\frac{-3y}{-3} = \frac{4-4x}{-3}$

$y = \frac{4-4x}{-3} = \frac{4(1-x)}{-3}$

x	1	-2	4	7	10
y	0	-4	4	8	12

(20) $W = 25t - 1250$



(a)

t	W
80	750
90	1000

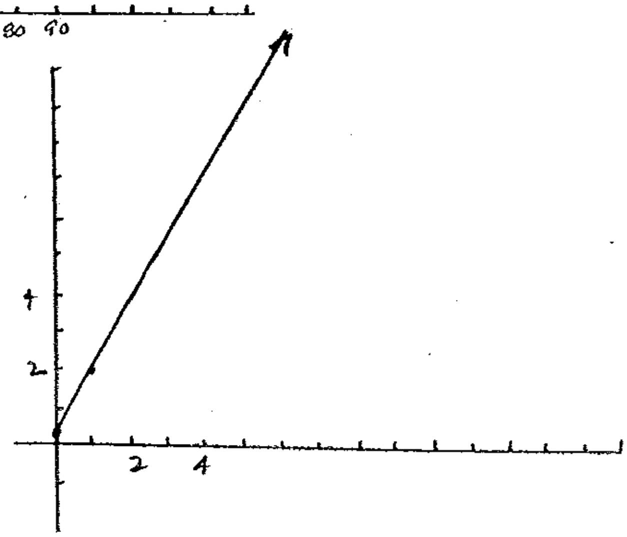
$\frac{\Delta W}{\Delta t}$

(b) $25 = m$

For increase in temperature of 1° chickens drink an additional 25 gallons of water per hour.

(c) $W = 25(100) - 1250 = 1250$ gallons/hr

(a)



(22) $W = 1.7x + 0.3$

(b) $m = 1.7$

For each increase of 1 foot in length of pipe the weight increases by 1.7 pounds.

(c) $W = 1.7(8) + 0.3 = 13.9$ pounds

(d) $19.85 = 1.7x + 0.3$

$19.55 = 1.7x$

$x = 11.5$ ft

(24)

(a)

t	V
0	26,500
1	24,800

$m = \frac{24,800 - 26,500}{1 - 0} = -1700$

$V = -1700t + 26,500$

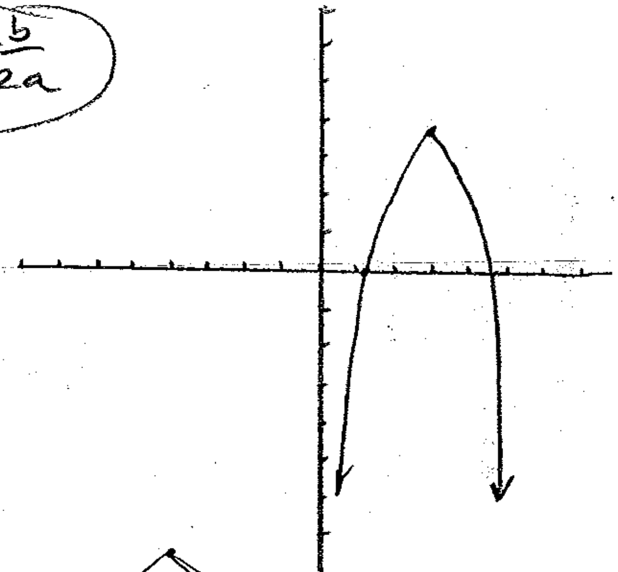
(b) $500 = -1700t + 26,500$

$-26,000 = -1700t$

$t = 15.3$ yrs.

Section 6.2

$x = \frac{-b}{2a}$



(8) $y = -x^2 + 6x - 5.5$

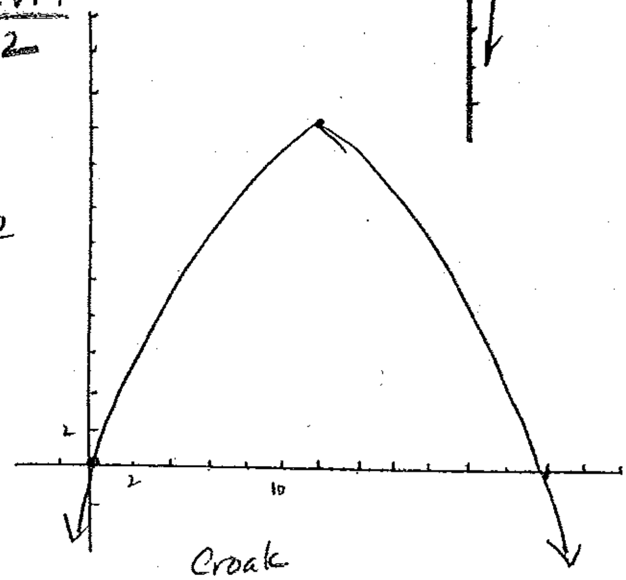
$x = \frac{-6}{-2} = 3 \qquad y = -(3)^2 + 6(3) - 5.5$
$= -9 + 18 - 5.5 = 3.5$

$V(3, 3.5)$

$-x^2 + 6x - 5.5 = 0$

$x = \frac{-6 \pm \sqrt{6^2 - 4(-1)(-5.5)}}{2(-1)} = \frac{-6 \pm \sqrt{14}}{-2}$

$x = 1.13, \ x = 4.87$



(10) $h = -\frac{1}{8}d^2 + 3d \qquad 0 \le d \le 12$

(a)

d	h
0	0
12	18

$d = \frac{-3}{2(-\frac{1}{8})} = 12$

(b) $h = -\frac{1}{8}(12)^2 + 3(12) = 18$ ft

Rivet

Croak

(12) $y = -\frac{1}{6}x^2 + 2x$

$x = \frac{-2}{2(-\frac{1}{3})} = 3$

$y = -\frac{1}{6}(3)^2 + 2(3) = \frac{9}{2}$ ft

$-\frac{1}{6}x^2 + 2x = 0$

$x = \frac{-2 \pm \sqrt{2^2 - 4(-\frac{1}{6})(0)}}{2(-\frac{1}{6})} = \frac{-2 \pm 2}{-\frac{1}{3}} = 12$

$y = -\frac{1}{2}x^2 + 4x$

$x = \frac{-4}{2(-\frac{1}{2})} = 4$

$y = -\frac{1}{2}(4)^2 + 4(4) = 8$ ft

$-\frac{1}{2}x^2 + 4x = 0$

$x = \frac{-4 \pm \sqrt{4^2 - 4(-\frac{1}{2})(0)}}{2(-\frac{1}{2})} = 8$ ft

(a) Croak, 8 ft

(b) Rivet, 12 ft

(14) (a)

x	y
0	20
2	22
4	20

$h = ad^2 + bd + c$

$20 = c$

$22 = 4a + 2b + c$

$20 = 16a + 4b + c$

$2 = 4a + 2b$

$0 = 16a + 4b$

$0 = -8a - 2b$

$2 = -4a$

$a = -\frac{1}{2}$

$\cancel{22} = -2 + 2b + 20$

$2b = 4 \quad b = 2$

$h = -\frac{1}{2}d^2 + 2d + 20$

(b) $0 = -\frac{1}{2}d^2 + 2d + 20$

$d = \dfrac{-2 \pm \sqrt{2^2 - 4(-\frac{1}{2})(20)}}{2(-\frac{1}{2})} = \dfrac{-2 \pm \sqrt{4+40}}{-1} = \dfrac{-2 \pm \sqrt{44}}{-1} = 8.63\,ft$

(16) $y = \dfrac{-32x^2}{V_0} + x$

(a) $y = \dfrac{-32x^2}{(192)^2} + x = \dfrac{-x^2}{1152} + x$

(b) $x = \dfrac{-1}{2(-\frac{1}{1152})} = \cancel{586}\ 576\,ft$

$y = \dfrac{-(586)^2}{1152} + 586 = 288\,ft$

(c) $-\dfrac{x^2}{1152} + x = 0$

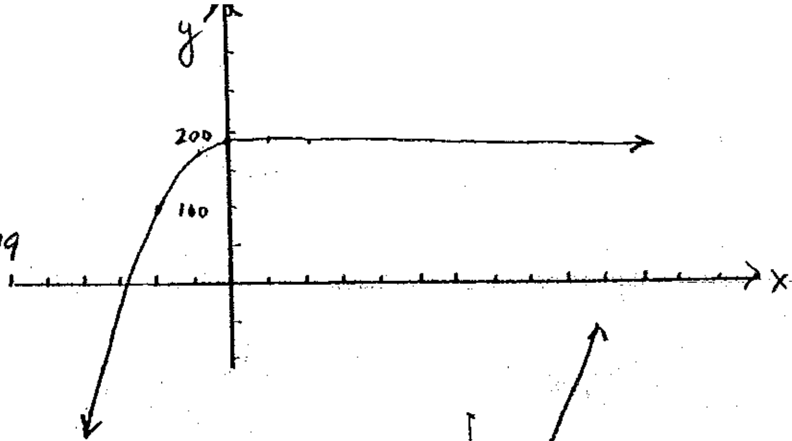
$x = \dfrac{-1 \pm \sqrt{1 - 4(\frac{-1}{1152})(0)}}{2(-\frac{1}{1152})} = \dfrac{-2}{\frac{-2}{1152}} = 1152\,ft$

? (d) 288 ft why?

Section 6.3

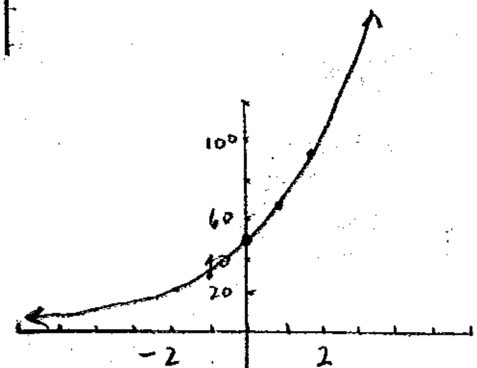
(10) $y = 200 - 10^{-x}$

x	-2	-1	0	1	2
y	100	190	199	199.9	199.99



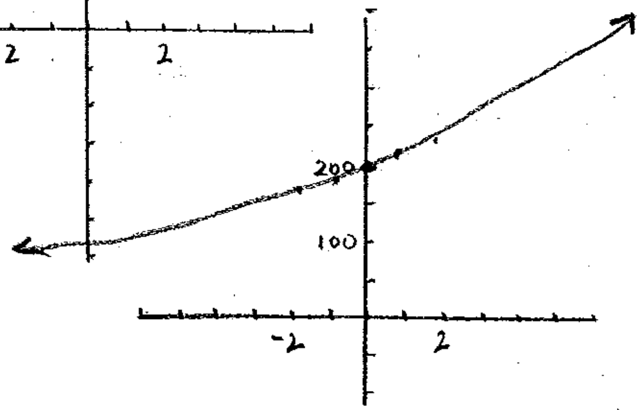
(8) $y = 50e^{0.3x}$

x	-2	-1	0	1	2
y	27.4	37.0	50	67.5	91.1



(12) $y = 200(1.015)^{4x}$

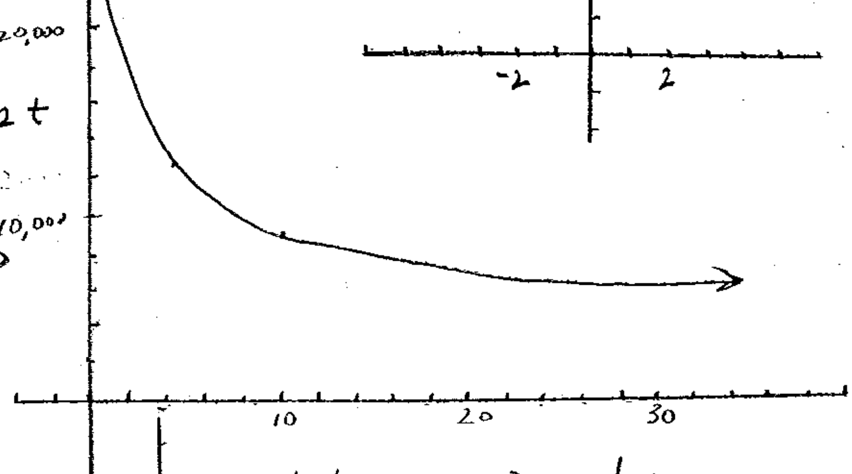
x	-2	-1	0	1	2
y	177.5	188.4	200	212.3	225.3



(14) $P = 6000 + 20{,}000(3)^{-0.2t}$

(a)

t	0	1	5	10	25
P	26,000	22,055.93	12,667.67	8222.22	6082.30



(b) $P(25) = \$6082.30$

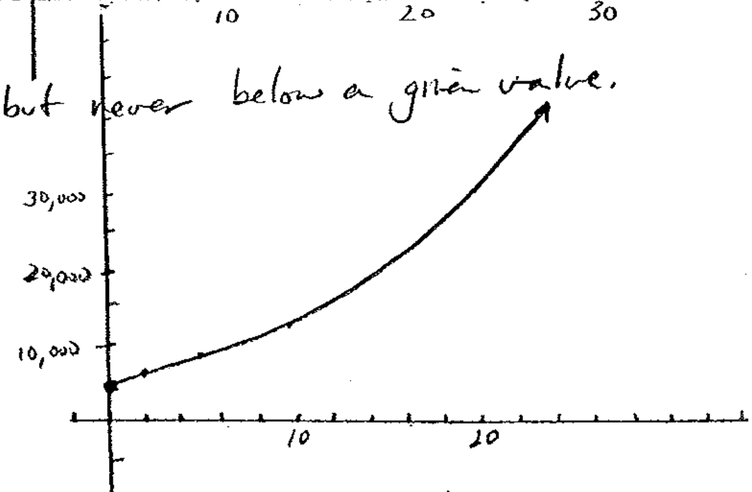
(c) Always greater than \$6000.

(d) Reasonable, profit will drop off but never below a given value.

(16) (a) $A = 5000e^{0.09t}$

(b)

t	0	1	5	10	20
A	5000	5470.87	7841.56	12,298.02	30,248.24



(c) \$30,248.24

(18) Temporary job, on the 21st day you will earn \$10,485.76

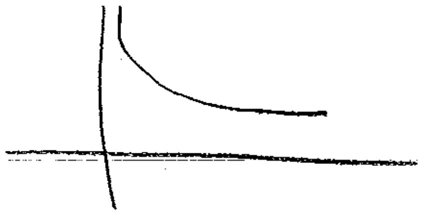
(20) $P = 48{,}898{,}000e^{.0245t}$

$P(14) = 48{,}898{,}000e^{.0245(14)}$

$\approx 68{,}905{,}534$

$K = \frac{49.9}{1000} - \frac{25.4}{1000} = .0245$

(22) Scores would decrease rapidly at first then level off



(24) $A = 1071.2 - 663.7e^{-0.205t}$

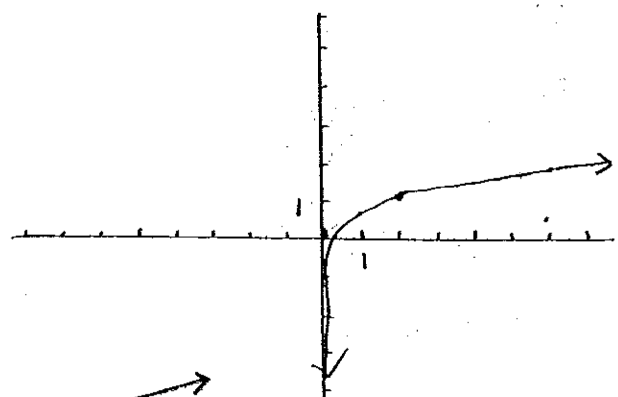
t	0	2	4	6	8
A	21	630.7	778.9	877.2	942.46

#23 more accurate at t=0, t=2

Section 6.4

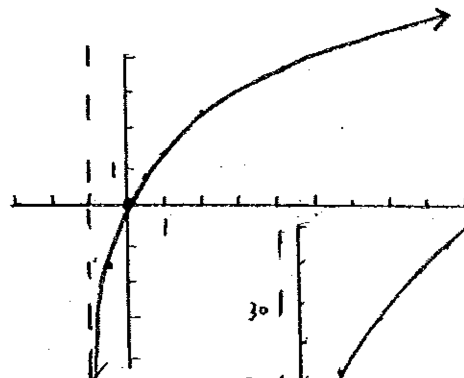
(8) $y = \log 5x$

x	1	2	5	10
y	.7	1	1.4	1.7



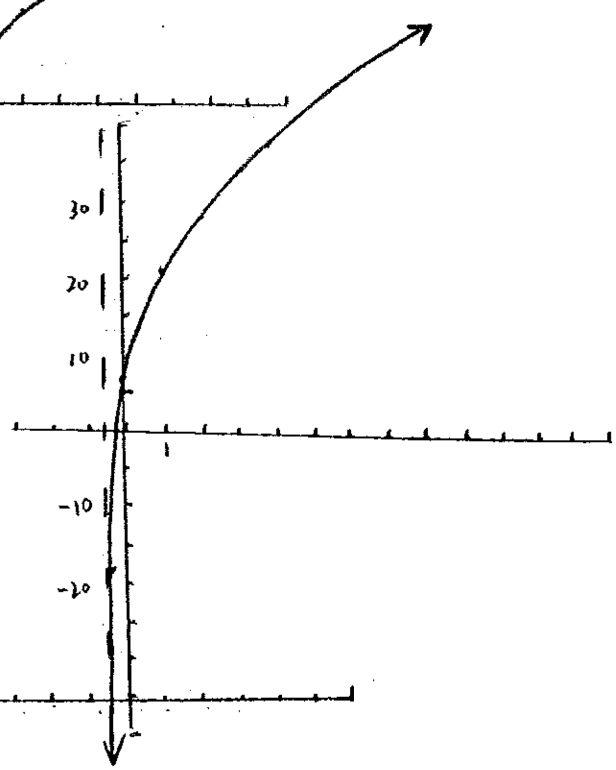
(10) $y = 5.3 \log(x+1)$

x	0	1	2	4	.5	-.5
y	0	1.6	2.5	3.7	.9	-1.6



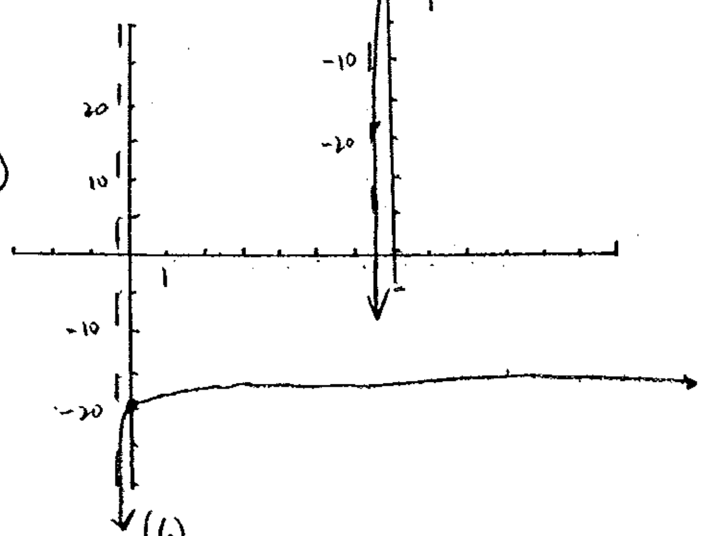
(12) $y = 5 + 14.3 \ln(2x + 1.2)$

x	-.5	0	1	2	4
y	-18.0	7.6	21.6	28.6	36.7



(14) $y = -20 + \log(3x + 1)$

x	0	1	2	4	10
y	-20	-19.4	-19.2	-18.89	-18.5

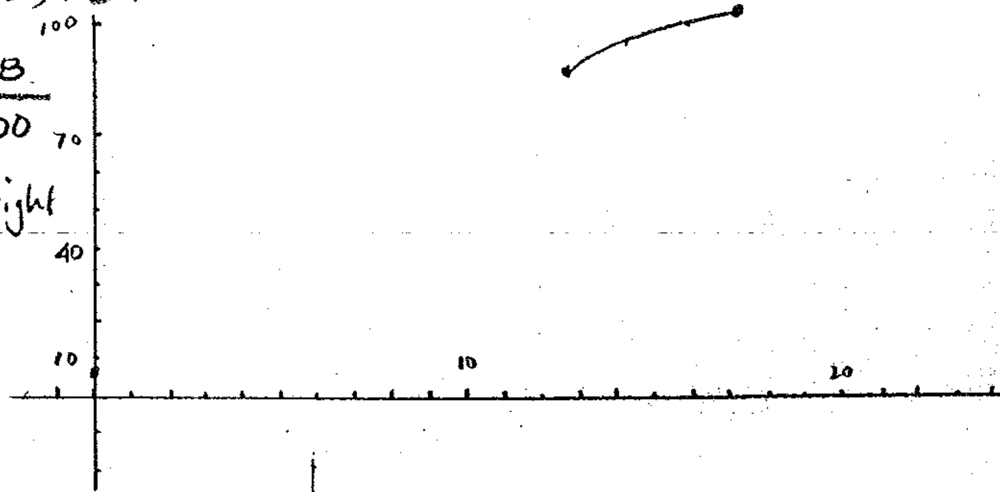


(16) $P = 16.7 \log(A-12) + 87$

(a)

A	13	15	17	18
P	87	95	98.7	100

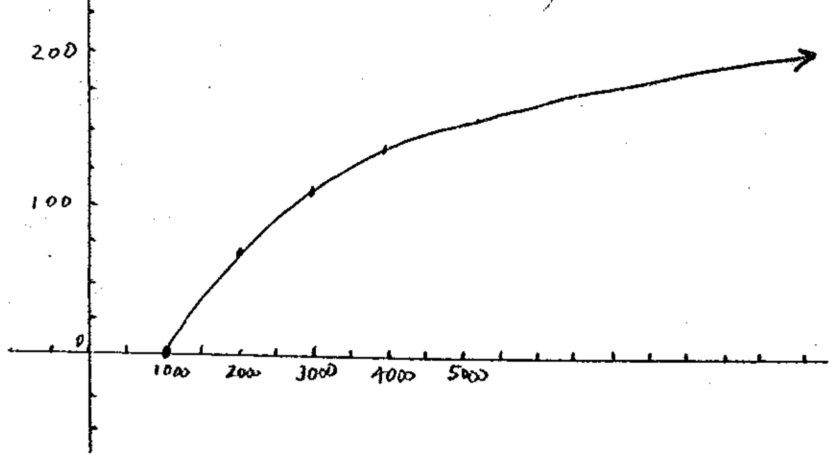
(b) Near 100% of adult height



(18) $n = -694.2 + 231.4 \log A$ $\quad A \geq 1000$

(a)

A	1000	2000	3000	4000	5000
n	0	69.7	110.4	139.3	161.7



(b) $n = -694.2 + 231.4 \log 1{,}000{,}000$

$n = 694.2$ months or 57.85 yrs

(20) $y = 27.4 \ln x - 170.3$

(a) $y = 27.4 \ln 1000 - 170.3 = 18.97 \approx 19$ years

(b) $y = 27.4 \ln 5000 - 170.3 = 63.07 \approx 63.1$ yrs

(22) (a) $y = a + b \ln x$

$42.1 = a + b \ln 1$

$42.1 = a$

$51.0 = 42.1 + b \ln 3$

$8.9 = b \ln 3$

$b = 8.10$

$y = 42.1 + 8.1 \ln x$

(b) $y = 42.1 + 8.1 \ln 13 = 62.9\%$

(c) $75 = 42.1 + 8.1 \ln x$

$32.9 = 8.1 \ln x$

$\ln x = 4.0617$

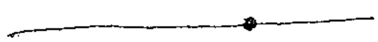
$x = e^{4.0617} \approx 58$

$1997 + 58 = 2055$

7.3

(2) The Greeks believed that the Golden ratio was more pleasing to the eye than any other ratio. It was used extensively in Greek architecture and art.

(4) Use the fact that $\frac{a+b}{a} \approx 1.62$ and $a+b$ is the length of the entire segment to solve for a.

(10) 

(14) $l \approx 1.5$ in $\quad w \approx .75$ in $\quad \frac{1.5}{.75} = 2$, so this is not a Golden rectangle

+1

(16) $l \approx 2.75$ in $\quad w \approx 1.75$ in $\quad \frac{2.75}{1.75} \approx 1.57$, so this is approximately a Golden rectangle

(18) If $l = 45$ mi, $\quad \frac{45}{w} \approx 1.62$ so $w \approx 2.7$

If $w = 4.5$ mi, $\quad \frac{l}{4.5} \approx 1.62$ so $l \approx 7.29$

+1

(22) If $l = 10\,3/4$ in, $\quad \frac{10.75}{w} \approx 1.62$ so $w \approx 6.64$

If $w = 10\,3/4$ in, $\quad \frac{l}{10.75} \approx 1.62$ so $l \approx 17.42$

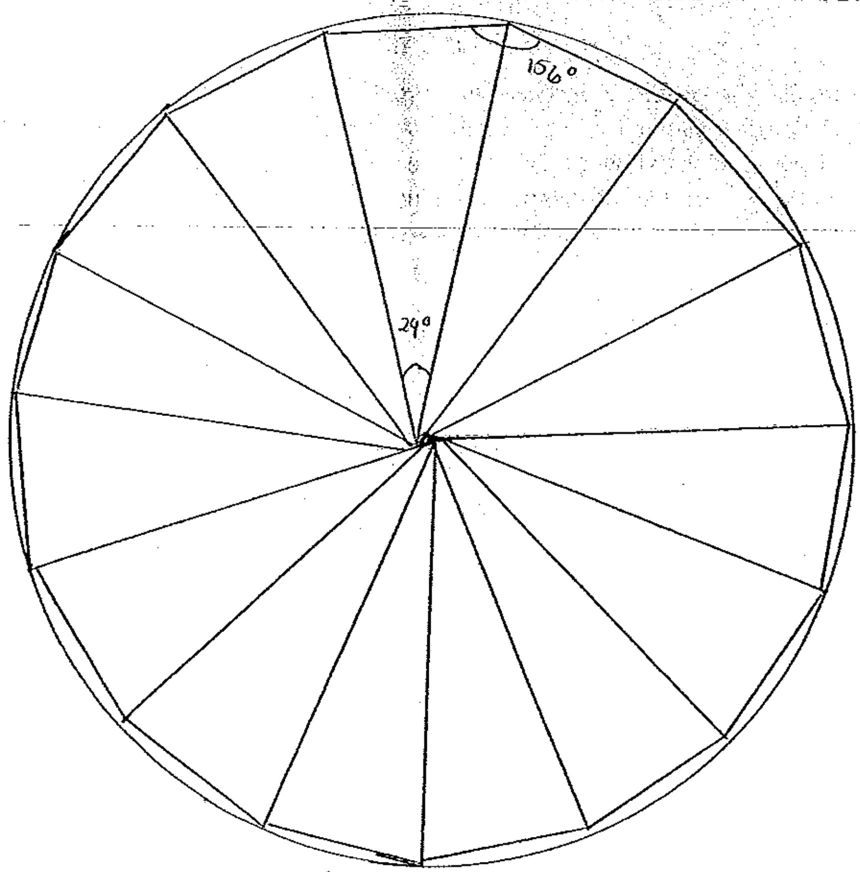
(24) For a Golden Cross we need $\frac{h}{w} \approx 1.62 \approx \frac{b}{t}$

If $h = 72$ in $\quad \frac{72}{w} \approx 1.62$ so $w \approx 44.4$ in

Also note $t+b = h$ so $b = h-t = 72-t$ which gives $\frac{72-t}{t} \approx 1.62$ or $72 \approx 2.62t$ giving $t \approx 27.48$ and then $1.62 \approx \frac{b}{27.48}$ so $b \approx 44.52$

(34) The measure of each interior angle in a regular 15-gon is $\frac{180(15-2)}{15} = \frac{180(13)}{15} = 156°$

+1

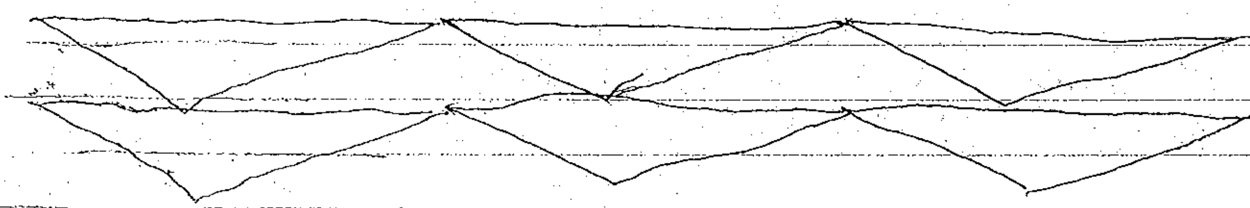


7.5

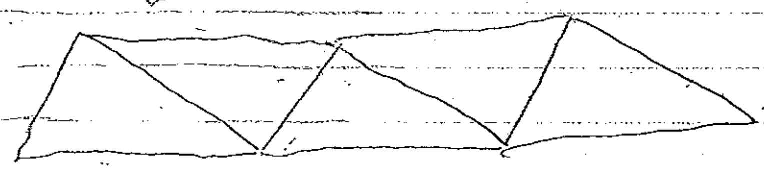
(4) The interior angles of the polygons must add to $360°$.

(12) The interior angle of a regular nonagon has measure $140°$, which does not evenly divide $360°$, so the shapes would overlap or leave a gap.

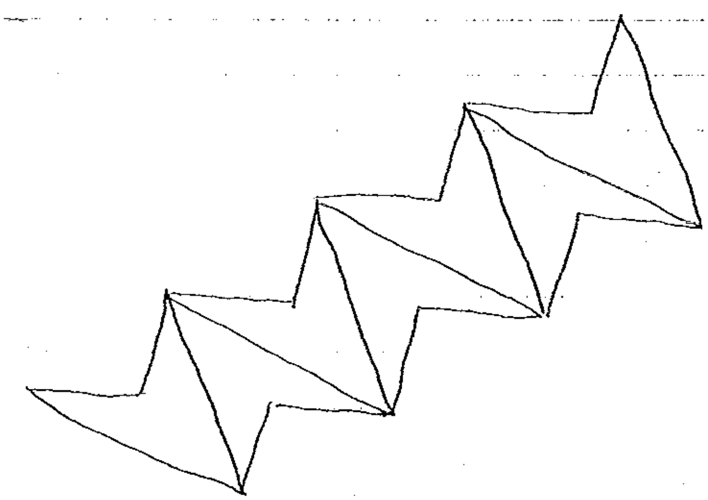
(16)



(18)

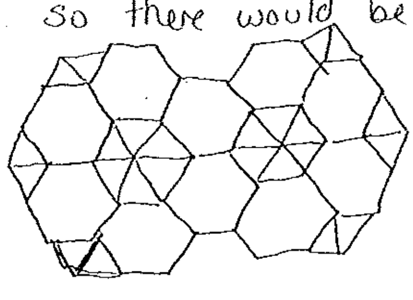


(24)



(26) The sum of the measures of the interior angles is $120° + 108° + 90° = 318$ so there would be a gap.

(28)



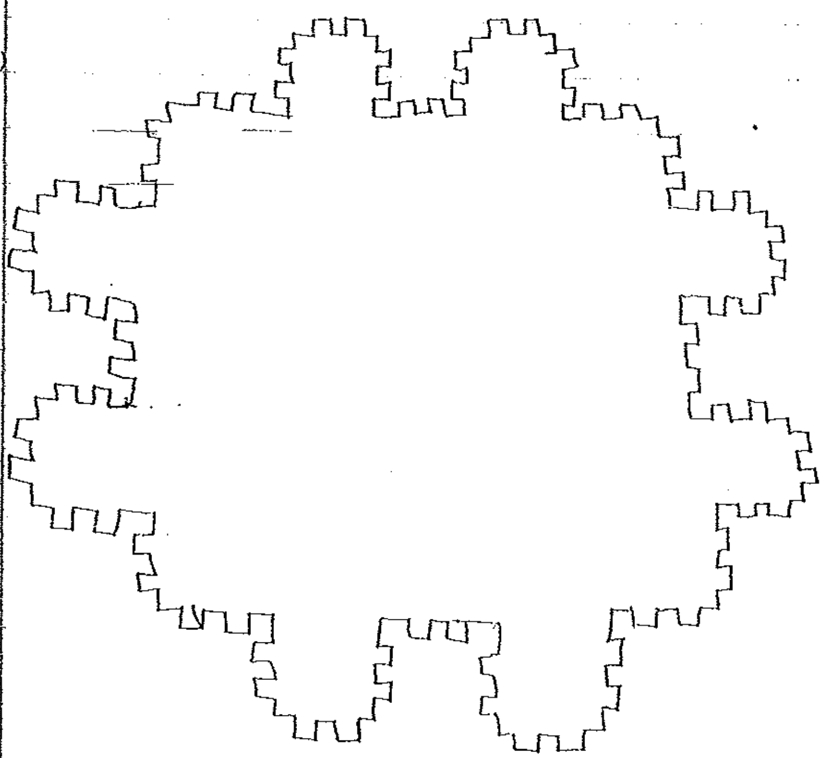
7.6

(2) The dimension of a fractal is given by $d = \frac{\log N}{\log(^1/_r)}$ where
r = the ratio of the length of the new object to the length of the original object
N = the number of new objects

(4) It means that is an object with dimension between 2 dimensions and 3 dimensions. From the formula we know that $2.6 \approx \frac{\log N}{\log(^1/_r)}$.

So for example, if $N=5$ at each step, we would have $r = .538$ so you would approximating have half the original length.

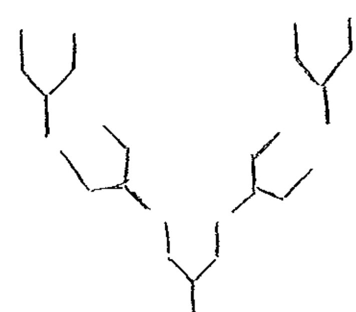
(10)



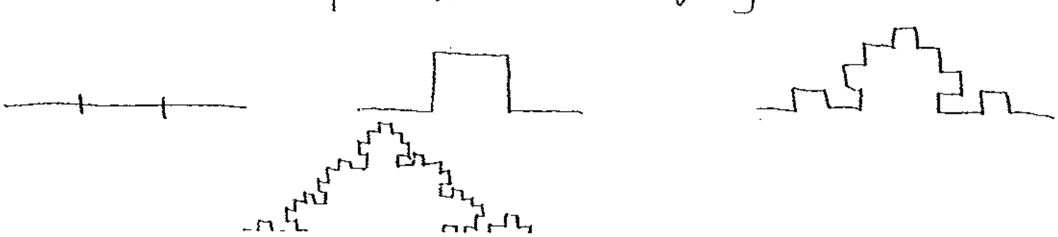
+2

(12)

(14)



(18) Just one example answers will vary



+2

(20) $N=7$ $r = 1/5$
$d = \frac{\log 7}{\log 5} \approx 1.21$

(22) $N=5$ $r = 1/4$
$d = \frac{\log 5}{\log 4} \approx 1.16$