MA 714: Topics in Discrete Mathematics
Experiments and Computations in Combinatorics and Algebra[1]
Spring 2018

1. General Information

Prof. Benjamin Braun
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Email: benjamin.braun@uky.edu
Office Phone: 257-6810
Class Time/Location: 12:00-12:50PM, MWF, CB 341
Office Location/Hours: 831 POT, by appointment (just stop by whenever the door is open or send me an email to set up a time)

2. Course Description, Expectations, and Grades

2.1. **Description.** The role of computer experimentation in mathematics is growing rapidly. In this course, all participants will increase their ability to effectively use computational tools for mathematical experiments in combinatorics and algebra. The following are the main goals for the course:

- develop familiarity with Sagemath and Macaulay2
- develop familiarity with basic aspects of Python, e.g. assignments, functions, data structures, control structures, exceptions and assertions
- gain an understanding of the historical and contemporary roles of experimentation in mathematics research
- develop good habits of computational problem solving
- develop familiarity with important open problems in combinatorics and algebra

This course will operate as a collaborative seminar based on problems and readings. Participants will be expected to complete course readings, participate in class discussions, give short presentations, and write code to explore/solve problems.

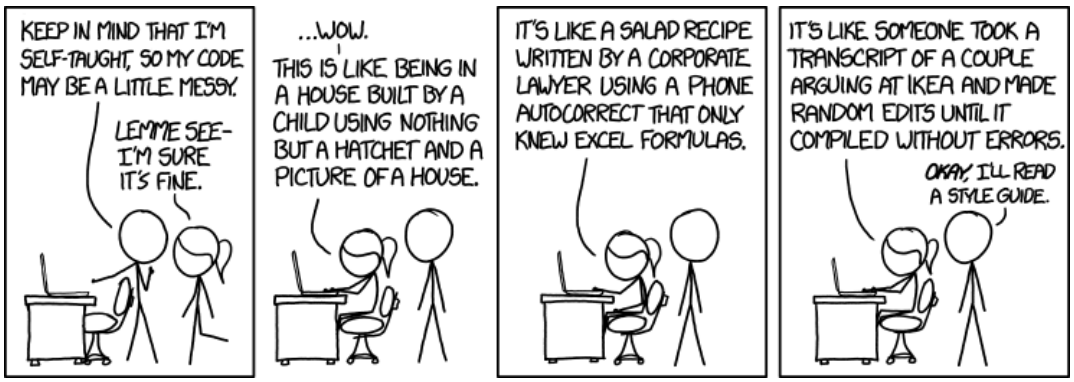

FIGURE 1. https://xkcd.com/1513/

[1]I reserve the right to change or amend this syllabus at any time for any reason.

2.2. **Expectations.**

- Students are expected to attend all classes. If students need to miss class for a legitimate reason, they are expected to let me know ahead of time.
- The course will be discussion- and problem-based. Thus, we will take turns presenting on different topics, sharing code to solve problems, providing each other with interesting examples, debating the merits of various approaches, etc.
- There will be regular reading assignments and coding exercises assigned. Students are expected to complete all assignments.
- It is expected that these readings and assignments will be completable in 6 hours per week outside of class time (2 hours per class session). If you are investing this amount of time and are unable to complete the readings and assignments, let me know right away.
- All students are expected to work collaboratively to cultivate a supportive and collegial environment.
- Feel free to bring your lunch. Noon is prime eating time.

2.3. **Grades.** Your total grade will be based on your overall engagement in the course and sufficiently meeting the course expectations listed above. If for any reason I feel that you are earning less than an "A" in the course, I will immediately send you a written message via email stating your current letter grade and the expectations required for raising it to an "A".

3. Texts and Computing Resources

3.1. **Texts.**

- *The Computer as Crucible: An Introduction to Experimental Mathematics*, by J. Borwein and K. Devlin
- *Mathematical Computation with SageMath*, available at

  https://members.loria.fr/PZimmermann/sagebook/english.html
- "Two Thousand Years of Combinatorics," by Don Knuth, in *Combinatorics: Ancient and Modern*, eds. Robin Wilson and John J. Watkins
- "Combinatorial tasks and outcome listing: Examining productive listing among undergraduate students," by Elise Lockwood and Bryan R. Gibson. *Educ Stud Math* (2016) 91:247-270
- "Exploratory Experimentation and Computation", by D. Bailey and J. Borwein, *Notices of the American Mathematical Society*, Volume 58, Number 10, November 2011. Available at

  http://www.ams.org/notices/201110/rtx111001410p.pdf
- And any other references that we run into that look good.

3.2. **Computing Resources.**

- **All students are required to create an account at CoCalc.com**, which is the web-based version of SageMath. This is for three reasons:
  - We will use CoCalc.com during class to demonstrate computations and share code in simultaneously-editable notebooks.
  - It will often be easier to use SageMath to interface with software such as LattE, Macaulay2, TOPCOM, GAP, etc than to install all of these programs locally on your machine. At CoCalc.com, this software is all automatically available through the SageMath notebooks.
  - I will use CoCalc.com to collect assignments.
- For any students who intend to do most of their computation at CoCalc.com rather than on their local machine, it is strongly recommended that you pay for an upgrade from the

free server to a member server. More information about paid subscriptions will be provided during the course.

4. Possible Topics We Will Discuss

There are three major areas that we will think about: building programming skills in SageMath/Python and Macaulay2, cultural and historical perspectives on experimentation in mathematics, and experimenting with open problems in combinatorics and algebra. Our emphasis on each of these, and the specific topics we explore, will develop dynamically during the course of the semester. The following list contains *possible* topics for exploration — we will not have time to discuss or work on all of these.

4.1. **Topics in SageMath/Python/Macaulay2 Programming.** We will need to develop basic skills with regard to coding, both regarding learning how to write code and regarding selecting algorithms in an informed manner.

(1) Sage/Python/Macaulay2 as a calculator, comparison commands
(2) assignments/bindings, sharing vs duplication
(3) data structures and types
    - lists, tuples, Booleans, strings, dictionaries, sets
    - methods for these types
(4) loops and conditionals, e.g. for, if, and while
(5) functions
    - snake_case convention
    - defined via both def and lambda commands
    - recursive vs iterative functions, limited aspects of dynamic programming
(6) file handling and input/output
(7) plotting with pylab
(8) exceptions and assertions
(9) classes and object-oriented programming

4.2. **Topics in Cultural and Historical Perspectives on Experimentation.** While our common mythology represents pure mathematics as an elegant framework of rigorously established results, the reality of mathematics is much more complicated and subtle, blending rigorous proofs, experimentation, questionable methods, and published results that turn out to be false.

(1) Contemporary context for "experimental" mathematics: chapter 1 of "The Computer as Crucible" and the Bailey/Borwein article "Exploratory Experimentation and Computation"
(2) Historical examples of proofs or observations that would now be considered experimental in nature
    - Newton's derivation of power series expansions for $\sin x$ and $\cos x$.
    - Euler's counterexample to primality of Fermat numbers.
    - Gauss's conjecture of the Prime Number Theorem
(3) The historical and contemporary role in combinatorics of systematic listing
    - Use of systematic listing over past 2000 years: Don Knuth article "Two Thousand Years of Combinatorics"
    - Role of systematic listing in successful undergraduate combinatorial problem solving: Elise Lockwood's model for combinatorial reasoning

4.3. **Problems in Combinatorics and Algebra.** When we write code and do comptuations, our focus will be experimenting to investigate the following (mostly unsolved) problems.

(1) Exercises to develop programming skills: projecteuler.net
(2) Open problems in number theory
    - Collatz Conjecture

- Erdős-Strauss conjecture
- Riemann Hypothesis (Lagarias's reformulation)

(3) Open problems regarding integer partitions
- Real-rootedness of Durfee polynomials
- Parity of the partition function

(4) Open problems regarding vertex and edge colorings of graphs
- Erdős-Faber-Lovasz conjecture
- Chromatic number for Hamming distance graphs, and more generally computing chromatic numbers of graphs
  - using roots of chromatic polynomials
  - as solutions to integer programming problems, with connections to fractional chromatic numbers using LP relaxations of IP problems
  - using Gröbner bases and coloring ideals
  - lower bounds via connectivity of neighborhood complexes
- Characterizing graphs with fixed-degree Nullstellensatz certificates for non-colorability using coloring ideals

(5) Open problems regarding topology of graph complexes
- Order of torsion elements in matching and chessboard complexes
- Homology of simplicial complexes related to Ferrers boards
- Torsion in independence complexes for grid graphs

(6) Open problems related to matchings in graphs
- Ahroni-Charbit-Howard conjecture that every k matchings of size $k+1$ in a bipartite graph possess a rainbow matching of size k
  - topological approaches using connectivity
  - matching numbers via integer programming

(7) Open problems regarding linear algebraic properties of graphs and simplicial complexes
- Duval-Reiner conjecture regarding majorization of Laplacian spectra for simplicial complexes, generalizing Grone-Merris conjecture/theorem
- Jacobs-Trevisan conjecture regarding halving point for Laplacian eigenvalues of trees
- What relationship, if any, is there between the algebraic connectivities of two trees and their respective diameters?

(8) Open problems regarding lattice polytopes and rational cones
- Do all smooth polytopes admit regular unimodular triangulations? Equivalently, do they admit Gröbner bases with squarefree initial ideals?
- Do all smooth polytopes have the integer decomposition property, i.e. have their Hilbert basis at height 1 in the cone over the polytope?
- What properties of a lattice polytope P implies that the Ehrhart h^*-polynomial of P is unimodal or real-rooted?
- What is the volume of the nth Birkhoff polytope? What is its Ehrhart polynomial?
- Given the d-dimensional unit cube C_d, what is the minimum number of lattice simplices in C_d required to cover C_d? What is the minimal number of simplices in a triangulation of C_d without introducing new vertices? What is the gap between these values as a function of d?
- For interesting classes of s-lecture hall cones, characterize their Hilbert bases.

(9) Open problems regarding matroids
- Do all matroid polytopes admit regular unimodular triangulations? Equivalently, do they admit Gröbner bases with squarefree initial ideals?
- Do Ehrhart polynomials for matroid polytopes have positive coefficients?
- Stanley's pure O-sequence conjecture for h-vectors of matroid complexes

5. **Administrative Policies**

5.1. **Excused Absences.** Students need to notify the professor of absences prior to class when possible. Senate Rules 5.2.4.2 defines the following as acceptable reasons for excused absences: (a) serious illness, (b) illness or death of family member, (c) University-related trips, (d) major religious holidays, and (e) other circumstances found to fit reasonable cause for nonattendance by the professor.

Students anticipating an absence for a major religious holiday are responsible for notifying the instructor in writing of anticipated absences due to their observance of such holidays no later than the last day in the semester to add a class. Two weeks prior to the absence is reasonable, but should not be given any later. Information regarding major religious holidays may be obtained through the Ombud (859-257-3737, http://www.uky.edu/Ombud/ForStudents_ExcusedAbsences.php.

Students are expected to withdraw from the class if more than 20% of the classes scheduled for the semester are missed (excused) per University policy.

Per Senate Rule 5.2.4.2, students missing any graded work due to an excused absence are responsible: for informing the Instructor of Record about their excused absence within one week following the period of the excused absence (except where prior notification is required); and for making up the missed work. The professor must give the student an opportunity to make up the work and/or the exams missed due to an excused absence, and shall do so, if feasible, during the semester in which the absence occurred.

5.2. **Verification of Absences.** Students may be asked to verify their absences in order for them to be considered excused. Senate Rule 5.2.4.2 states that faculty have the right to request appropriate verification when students claim an excused absence because of illness, or death in the family. Appropriate notification of absences due to University-related trips is required prior to the absence when feasible and in no case more than one week after the absence.

5.3. **Academic Integrity.** Per University policy, students shall not plagiarize, cheat, or falsify or misuse academic records. Students are expected to adhere to University policy on cheating and plagiarism in all courses. The minimum penalty for a first offense is a zero on the assignment on which the offense occurred. If the offense is considered severe or the student has other academic offenses on their record, more serious penalties, up to suspension from the University may be imposed.

Plagiarism and cheating are serious breaches of academic conduct. Each student is advised to become familiar with the various forms of academic dishonesty as explained in the Code of Student Rights and Responsibilities. Complete information can be found at the following website: http://www.uky.edu/Ombud. A plea of ignorance is not acceptable as a defense against the charge of academic dishonesty. It is important that you review this information as all ideas borrowed from others need to be properly credited.

Senate Rules 6.3.1 (see http://www.uky.edu/Faculty/Senate/ for the current set of Senate Rules) states that all academic work, written or otherwise, submitted by students to their instructors or other academic supervisors, is expected to be the result of their own thought, research, or self-expression. In cases where students feel unsure about a question of plagiarism involving their work, they are obliged to consult their instructors on the matter before submission.

When students submit work purporting to be their own, but which in any way borrows ideas, organization, wording, or content from another source without appropriate acknowledgment of the fact, the students are guilty of plagiarism.

Plagiarism includes reproducing someone else's work (including, but not limited to a published article, a book, a website, computer code, or a paper from a friend) without clear attribution. Plagiarism also includes the practice of employing or allowing another person to alter or revise the work, which a student submits as his/her own, whoever that other person may be. Students may

discuss assignments among themselves or with an instructor or tutor, but when the actual work is done, it must be done by the student, and the student alone.

When a student's assignment involves research in outside sources or information, the student must carefully acknowledge exactly what, where and how he/she has employed them. If the words of someone else are used, the student must put quotation marks around the passage in question and add an appropriate indication of its origin. Making simple changes while leaving the organization, content, and phraseology intact is plagiaristic. However, nothing in these Rules shall apply to those ideas, which are so generally and freely circulated as to be a part of the public domain.

Please note: Any assignment you turn in may be submitted to an electronic database to check for plagiarism.

5.4. **Accommodations due to disability.** If you have a documented disability that requires academic accommodations, please see me as soon as possible during scheduled office hours. In order to receive accommodations in this course, you must provide me with a Letter of Accommodation from the Disability Resource Center (DRC). The DRC coordinates campus disability services available to students with disabilities. It is located on the corner of Rose Street and Huguelet Drive in the Multidisciplinary Science Building, Suite 407. You can reach them via phone at (859) 257-2754 and via email at drc@uky.edu. Their web address is http://www.uky.edu/DisabilityResourceCenter.

5.5. **Non-Discrimination Statement and Title IX Information.** The University of Kentucky faculty are committed to supporting students and upholding the University's non-discrimination policy.

Discrimination is prohibited at UK. If you experience an incident of discrimination we encourage you to report it to Institutional Equity & Equal Opportunity (IEEO) Office, 13 Main Building, (859) 257-8927.

Acts of Sex- and Gender-Based Discrimination or Interpersonal Violence: If you experience an incident of sex- or gender-based discrimination or interpersonal violence, we encourage you to report it. While you may talk to a faculty member or TA/RA/GA, understand that as a "Responsible Employee" of the University these individuals MUST report any acts of violence (including verbal bullying and sexual harassment) to the University's Title IX Coordinator in the IEEO Office. If you would like to speak with someone who may be able to afford you confidentiality, the Violence Intervention and Prevention (VIP) program (Frazee Hall Lower Level; http://www.uky.edu/StudentAffairs/VIPCenter/), the Counseling Center (106 Frazee Hall, http://www.uky.edu/StudentAffairs/Counseling/), and the University Health Services (http://ukhealthcare.uky.ed health/) are confidential resources on campus.