Abstract

On the Brunn Minkowski Inequality and a Minkowski Problem for Nonlinear Capacitiies

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In this seminar we discuss two classical problems in convex geometry. The first problem is a Brunn-Minkowski inequality for a nonlinear capacity, $\mathrm{Cap}_{\mathcal{A}}$:

$$[\mathrm{Cap}_{\mathcal{A}}(\lambda E_1 + (1-\lambda)E_2)]^{\frac{1}{(n-p)}} \geq \lambda\, [\mathrm{Cap}_{\mathcal{A}}(E_1)]^{\frac{1}{(n-p)}} + (1-\lambda)\, [\mathrm{Cap}_{\mathcal{A}}(E_2)]^{\frac{1}{(n-p)}}$$

when $1 < p < n, 0 < \lambda < 1$, and E_1, E_2 are convex compact sets with positive $\mathcal{A}$ capacity. In the second part of this talk we discuss a Minkowski existence problem for a certain measure associated with a compact convex set E with nonempty interior and its $\mathcal{A}$ harmonic capacitary function in the complement of E. If μ_E denotes this measure, then the Minkowski problem we consider in this setting, is that for a given finite Borel measure $\tilde{\mu}$ with support in the unit sphere of Euclidean n space, find necessary and sufficient conditions for which there exists E as above with $\mu_E = \tilde{\mu}$.