Orchestrating Discussions and other Big Ideas from Math Ed
Orchestrating Mathematical Discussions

1. Anticipating
2. Monitoring
3. Selecting
4. Sequencing
5. Connecting

What responses do you think precalculus students might give to the following problem?

The table at right lists values of $f(x)$ for various values of $x$. Determine $f^{-1}(3)$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-3</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>
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Consider the following questions in reference to the work samples:

- What aspects of each student’s response is mathematically relevant?

- What can you infer about each student’s mathematical thinking?

- Fun Fact: When a student does not “get it,” there is often a reason!
What is Professional Noticing (PN)?

Attending to salient aspects of children’s work

Interpreting children’s work in context of the mathematics

Deciding appropriate next steps, diagnostic and/or instructional

**Instantiation of noticing on one of the students’ work**

Consider the work of student 3:

<table>
<thead>
<tr>
<th>Attending</th>
<th>Interpreting</th>
</tr>
</thead>
</table>
| • Finds a valid rule for the information in the table  
• Correctly sets up an equation to solve for the inverse function  
• Correctly substitutes the value “3” for the right variable and identifies the requested value | • Student 3’s thinking about functions is likely dominated by formulas for functions  
• Student 3 has a good command of the formal operations involved with computing inverses for functions  
• Student 3 *may* not have good intuition about what the inverse of a function means |
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What set of responses do you think would generate a good mathematical discussion?

In what order should the responses be considered?

Relatedly, what mathematical content does this question presuppose? What mathematical ideas might come next?
What is a Learning Trajectory?

<table>
<thead>
<tr>
<th>Mathematical Goal</th>
<th>A big idea that is mathematically central, coherent, and consistent with students’ thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Developmental Progression</td>
<td>A description of a typical path students follow in developing mathematical skill and understanding</td>
</tr>
<tr>
<td>Sequence of Tasks</td>
<td>A set of instructional activities matched to each level described in the developmental progression</td>
</tr>
</tbody>
</table>

An overarching developmental progression of function understanding

1. **Action:**
   
   A function is a procedure for computation. Operating with functions is strictly confined to applying procedural rules.

2. **Process:**
   
   A function is a dynamic transformation of quantities. Meaning can be developed for function arithmetic, composition, and inverse.

3. **Object:**
   
   A function is an object that can be acted upon. Sense can be made of derivatives and integrals.

Dubinsky & McDonald (2001) APOS (just read the first three pages)
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How do the responses you selected connect to each other?

How can a conversation be structured to help students make those connections?
## What are Talk Moves?

<table>
<thead>
<tr>
<th>Talk Move</th>
<th>What it Means</th>
<th>Example Prompt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revoicing</td>
<td>The teacher restates using a question. Useful for clarifying language and eliciting more talk.</td>
<td>“So, you multiplied both sides by 2?”</td>
</tr>
<tr>
<td>Rephrasing</td>
<td>Asking students to restate someone else’s ideas in their own words.</td>
<td>“Who can share what Ricardo just said, but using your own words?”</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Asking students to analyze, interpret, or evaluate the work of another student</td>
<td>“Do you think Justin’s strategy will work for all functions or just linear ones? Why?”</td>
</tr>
<tr>
<td>Elaborating</td>
<td>This is a request for students to challenge, add on, elaborate, or give an example</td>
<td>“Can you give me an example?” “Do you see a connection between your idea and hers?”</td>
</tr>
<tr>
<td>Waiting</td>
<td>Waiting is good if students are thinking, but Waiting for Godot is not productive</td>
<td>“This question is important. Let’s take some time to think about it”</td>
</tr>
</tbody>
</table>

Let’s take another look...

1. Anticipating:
   - Reasoning about what your particular students know and can do

2. Monitoring:
   - Making sense of students’ mathematical productions

3. Selecting:
   - Deciding what is relevant to the mathematical (and instructional) context

4. Sequencing:
   - Reasoning about how students learn

5. Connecting:
   - Reasoning about what mathematics needs to be communicated and how
What is Mathematical Knowledge for Teaching (MKT)?

Hill & Ball (2009) The curious - and crucial - case of mathematical knowledge for teaching (use “View Now @UK” link)
Number Talks are an informal setting in which to have mathematical discussions

- A number talk is a short conversation around purposefully crafted computation problems
  - Students are asked to communicate their thinking and justify their solutions
  - An explicit goal of number talks is to develop and analyze a range of valid strategies
    - Which are most general? Most efficient? Most clear?

- Additional purposes of number talks include
  - Building a cohesive, respectful classroom community
  - Developing patterns and norms of classroom discourse
    - Sociomathematical Norms of [Cobb and Yackel (1996)]

Parrish (2011) [Number Talks Build Numerical Reasoning]
Teaching is Messy. The End.