A Symposium to Mark the Seventieth Birthday of Professor James C. Beidleman, to Honor his Mathematical Achievements, and to Celebrate his Friendship.

Program & Abstracts

Organizers | Sponsored by the University of Kentucky Mathematics Department

Ben Brewster
Lyn Lemieux
Matthew Ragland

All talks will be held in Ballroom #4.

Friday, September 8; Morning Session

8:00	A.M.	Breakfast
9:00-9:45	A.M.	**Derek Robinson (University of Illinois at Urbana-Champaign)** *Groups which Contain No HNN-Extensions*
9:50-10:25	A.M.	Coffee, snacks, and exchange of ideas
10:30-11:15	A.M.	**Martin Evans (University of Alabama)** *Free Abelianized Extensions of Groups*
11:20-11:55	A.M.	Exchange of ideas

End of Morning Session

Lunch at noon in the Clay Room

Friday, September 8; Afternoon Session

1:30-2:15	P.M.	**Martyn Dixon (University of Alabama)** *The Class of Groups Satisfying Min-p for all Primes p*
2:20-2:55	P.M.	Coffee, snacks, and exchange of ideas
3:00-3:45	P.M.	**Arnold Feldman (Franklin & Marshall College)** *Complements, 2-Maximal Subgroups, Fischer Subgroups, and Injectors*
3:50-4:25	P.M.	Exchange of ideas

End of Afternoon Session

Dinner from 6:00pm to 9:00pm in the Daniel Boone Room

Saturday, September 9; Morning Session

8:00	A.M.	Breakfast
9:00-9:45	A.M.	**Peter Hauck (Eberhard-Karls-Universität Tübingen)** *2-Generated Subgroups of Finite Groups*
9:50-10:25	A.M.	Coffee, snacks, and exchange of ideas
10:30-11:15	A.M.	**Ben Brewster (SUNY Binghamton)** *Embedding Properties in Direct Products*
11:20-11:55	A.M.	Exchange of ideas

End of Morning Session

Lunch at noon in the Breckinridge Room

Saturday, September 9; Afternoon Session

1:30-2:15	P.M.	**Tuval Foguel (Auburn University Montgomery)** *Groups and Loops with a Finite Covering by Isomorphic Abelian Subgroups*
2:20-2:55	P.M.	Coffee, snacks, and exchange of ideas
3:00-3:45	P.M.	**Matthew Ragland (Auburn University Montgomery)** *$\mathcal{T}$-groups and their Generalizations*
3:50-4:25	P.M.	Exchange of ideas

End of Afternoon Session

End of Symposium

Groups Which Contain No HNN-Extensions

DEREK ROBINSON, University of Illinois at Urbana-Champaign

Abstract

A group G is called HNN-free if G does not have a subgroup which is a non-trivial HNN-extension. This is equivalent to the property: $g \in G$ and $H^g \leq H$ implies $H^g = H$.

We will discuss the class of HNN-free groups and some of its interesting subclasses. We will prove that for many groups, including all those that are elementary amenable, HNN-free is the same as virtually polycyclic.

As an application, we show how to classify locally graded groups all of whose subgroups are pronormal.

Free Abelianized Extensions of Groups

MARTIN EVANS, University of Alabama

Abstract

Let G be a d-generator group and F_n an absolutely free group of rank $n \geq d$. Clearly there exist epimorphisms $\rho : F_n \to G$. On setting $K_\rho = \ker \rho$ for each such ρ, we obtain presentations $P_\rho : K_\rho \hookrightarrow F_n \twoheadrightarrow G$. The *free abelianized extension of* G *associated with* ρ is the group F_n/K'_ρ. Such free abelianized extensions of G form an interesting class of groups for study. For instance, we may ask "How does the isomorphism class of F_n/K'_ρ depend on ρ?" We'll discuss some "classical" results and some recent results that connect the subject to algebraic K-theory.

The Class of Groups Satisfying Min-p for all Primes p

MARTYN DIXON, University of Alabama

Abstract

A group G satisfies min-p for the prime p if every p-subgroup of G has the minimal condition. Thus the p-subgroups of G are Černikov groups. In this talk I will give a survey of results concerning the class $\mathfrak{X}$ of locally finite groups with min-p for all primes p. Among other things I'll discuss the structure of such groups and Sylow properties. I'll also discuss what is known concerning Fitting classes and formations and I will also show how to construct uncountable groups in the class.

Complements, 2-maximal subgroups, Fischer subgroups, and injectors

ARNOLD FELDMAN, Franklin & Marshall College

Abstract

All groups considered here are finite and solvable. A subgroup H is an *injector of* G if H is an $\mathcal{F}$-injector for some Fitting set $\mathcal{F}$ of G. We write $H \in \mathrm{inj}(G)$. A subgroup H is *2-maximal* in G if H is not maximal in G, and whenever $H < M < G$, M is maximal in G. If H is an injector of a finite solvable group G, then

(**) If X is subnormal in G, then $H \cap X$ is pronormal in $N_G(X)$.

However, a subgroup satisfying (**) need not be an injector [See Exercise 2, p.553 in Doerk and Hawkes' *Finite Soluble Groups*, for example.]

We prove that if H satisfies (**) in G and has a normal complement in G, then $H \in inj(G)$. Also, if H is 2-maximal in G and satisfies (**) in G, then $H \in \mathrm{inj}(G)$.

An early rival to the injector in the study of Fitting sets is the Fischer subgroup: If $\mathcal{F}$ is a Fitting set, U is a *Fischer $\mathcal{F}$-subgroup* of G if U contains every subgroup in $\mathcal{F}$ that U normalizes. We prove that if G is a finite solvable group and H is a 2-maximal Fischer $\mathcal{F}$-subgroup of G, then $H \in \mathrm{inj}(G)$.

This is joint work with Rex Dark and Maria Dolores Perez-Ramos.

2-Generated Subgroups of Finite Groups

PETER HAUCK, Eberhard-Karls-Universität Tübingen

Abstract

Let $\mathcal{L}$ be a class of finite groups. Two subgroups A and B of a group G are called $\mathcal{L}$-connected if $\langle a, b \rangle \in \mathcal{L}$ for all $a \in A$ and $b \in B$. This concept was introduced by A. Carocca in 1996 and has received considerable interest recently (for instance in a paper of J. Beidleman and H. Heineken from 2004).

The leading question for the talk is the following: What can be said about a finite group that is generated by two $\mathcal{L}$-connected subgroups? We present some answers to this question for various important classes $\mathcal{L}$ lying between $\mathcal{A}$, the class of all finite abelian groups, and $\mathcal{N}^2$, the class of all finite solvable groups of nilpotent length at most 2.

Embedding Properties in Direct Products

BEN BREWSTER, SUNY Binghamton

Abstract

The talk is to present an overview, and then some recent developments in the on-going project of determining which subgroups of a direct product possess an embedding property. The determination usually is in the terms of the projections onto the coordinates, the intersection with the coordinates and the embedding of the corresponding section; projections modulo the intersection. For example $U \triangleleft G_1 \times G_2$ if and only if $\pi_i(U), U \cap G_i \triangleleft G_i$ for $i = 1, 2$ and $\pi_i(U)/U \cap G_i \leq Z(G/U \cap G_i)$.

Recently some progress has been made on the embedding property "Satisfies the Frattini Argument". This is the condition $(7.\alpha)$ in *Finite Soluble Groups* by Doerk and Hawkes, and is intertwined with other embedding properties discussed in the latter sections of Chapter 1 in *Finite Soluble Groups*.

There is possible connection to injectors and, should time be sufficient, I hope to say some about this although it is not necessarily confined to the direct product situation.

Groups and Loops with a Finite Covering by Isomorphic Abelian Subgroups

TUVAL FOGUEL, Auburn University Montgomery

Abstract

In this talk we look at groups and loops with a finite covering by proper isomorphic abelian subgroups. In most of the talk, we will look at finite groups with such a covering. We will see that there are no simple groups with such a covering, but on the other hand, that every finite group is a summand of a group with such a covering. Examples of simple loops with such a covering will be discussed.

$\mathcal{T}$-groups and their Generalizations

MATTHEW RAGLAND, Auburn University Montgomery

Abstract

$\mathcal{T}$-groups are those groups G in which normality is a transitive relation, that is, those groups G for which $H \trianglelefteq K \trianglelefteq G$ always implies $H \trianglelefteq G$.

To generalize, one considers $\mathcal{PT}$-groups and $\mathcal{PST}$-groups. We call a subgroup H of a group G permutable (Sylow-permutable) in G provided $HK = KH$ for each subgroup (Sylow subgroup) K of G. That normal subgroups are permutable (Sylow-permutable) is clear from the definition of normality. So, the class of groups in which permutability (Sylow-permutability) is a transitive relation, the so called $\mathcal{PT}$-groups ($\mathcal{PST}$-groups), is a natural way to generalize the class of $\mathcal{T}$-groups.

A normal subgroup H of a group G is, of course, normalized by every element of the group. Suppose H is only normalized by the elements $g \in G$ such that $(|g|, |H|) = 1$. Call such a subgroup H R-normal in G. Similarly, consider the following definitions. H is an R-permutable (R-Sylow-permutable) subgroup of G if H permutes with all subgroups (Sylow-subgroups) K of G such that $(|K|, |H|) = 1$. What happens when we force these relations to be transitive? We hope to answer this question.

$H \leq G$ is said to be normal sensitive if the map $N \to H \cap N$ sends the lattice of normal subgroups of G onto the lattice of normal subgroups of H. A seemingly forgotten theorem of S. Bauman states the following:

Theorem. *Every subgroup $H \leq G$ is normal sensitive if and only if G is a solvable $\mathcal{T}$-group.*

Does this result extend nicely to $\mathcal{PT}$-groups as well as $\mathcal{PST}$-groups? We hope to answer this question as well.

The last part of this talk concerning sensitivity is joint work with Jim Beidleman.