ORAM 8
List of Abstracts

1. Plenary Speakers

Matthew Badger, University of Connecticut

Open problems in curves, sets, and measures

Abstract: This talk will serve as an introduction to recent work and open problems on the geometry of sets and measures in Euclidean space. One goal of this inquiry is to understand how general measures interact with canonical families of lower dimension sets such as rectifiable curves, Hölder curves, or Lipschitz surfaces. Along the way, one must develop tests to identify subsets of the canonical sets. Our tour will highlight classical theorems in topology, modern results in quantitative geometry, and recent advances in geometric measure theory and harmonic analysis with an emphasis on open problems. This is joint work with R. Schul and with L. Naples and V. Vellis.

Andreas Seeger, University of Wisconsin

Spherical maximal functions on the Heisenberg groups

Abstract: Let V be a hyperplane in the Heisenberg group $\mathbb{H}^n$ and μ the surface measure of a sphere in V. In the talk I will discuss old and new results on $L^p(\mathbb{H}^n)$-boundedness for the maximal operator $f \mapsto \sup_t |f * \mu_t|$ generated by the automorphic dilations.

Plamen Stefanov, Purdue University

Local and global boundary rigidity

Abstract: The boundary rigidity problem consist of recovering a Riemannian metric in a domain, up to an isometry, from the distance between boundary points. We show that in dimensions three and higher, knowing the distance near a fixed strictly convex boundary point allows us to reconstruct the metric inside the domain near that point, and that this reconstruction is stable. We also prove semi-global and global results under certain an assumption of the existence of a strictly convex foliation. The problem can be reformulated as a recovery of the metric from the arrival times of waves between boundary points; which is known as travel-time tomography. The interest in this problem is motivated by imaging problems in seismology: to recover the sub-surface structure of the Earth given travel-times from the propagation of seismic waves. In oil exploration, the seismic signals are man-made and the problem is local in nature. In particular, we can recover locally the compressional and the shear wave speeds for the elastic Earth model, given local information. The talk is based on joint work with G.Uhlmann (UW)

and A.Vasy (Stanford). We will also present results for a recovery of a Lorentzian metric from red shifts motivated by the problem of observing cosmic strings.

Catherine Sulem, University of Toronto

Surface water waves over a variable bottom

Abstract: We examine the effect of a periodic bottom on the free surface of a fluid linearized near the stationary state, and we develop a Bloch theory for the linearized water wave system. This analysis takes the form of a spectral problem for the Dirichlet-Neumann operator of the fluid domain with periodic bottom. We find that, generically, the presence of the bottom results in the splitting of double eigenvalues creating a spectral gap. (This is joint work with W. Craig, M. Gazeau, and C. Lacave).

Sijue Wu, University of Michigan

On the motion of two dimensional water waves with angled crests

Abstract: We will discuss recent progress on the study of two dimensional water waves with angled crests.

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2. Contributed Speakers

Jack Arbunich, University of Illinois at Chicago (Session 2D)

Regularizing nonlinear Schrödinger equations through partial off-axis variations

Abstract: We study a class of (focusing) nonlinear Schrödinger-type equations derived recently by Dumas, Lannes and Szeftel within the mathematical description of high intensity laser beams. These equations incorporate the possibility of a (partial) off-axis variation of the group velocity of such laser beams through a second order partial differential operator acting in some, but not necessarily all, spatial directions. We investiagate the initial value problem for such models and obtain global well-posedness in L^2-supercritical situations, even in the case of only partial off-axis dependence.

Charles Baker, Ohio State University (Session 3D)

Localization of eigenvalues of doubly cyclic matrices

Abstract: For fixed positive α and β, and a fixed integer n, $n \geq 2$, we consider the family of matrices $\text{diag}(a_1, a_2, \ldots, a_n) - \text{diag}(b_1, b_2, \ldots, b_n)\Sigma_*$, where all the a_k's and b_k's are positive, the geometric mean of the a_k's and b_k's must be α and β, respectively, and Σ_* denotes the permutation matrix corresponding to the cycle $(1, 2, \ldots, n)$.

C. Johnson, Z. Price, and I. Spitkovsky conjectured that in this family, the number of eigenvalues in the left half-plane is maximized by $\alpha I - \beta\Sigma_*$; we prove this conjecture. Moreover, the complete range of possibilities for the number of eigenvalues in the left half-plane is demonstrated: if $\alpha < \beta$, then any odd number between 1 and the maximum, inclusive, is attainable.

This work is joint with B. Mityagin (Ohio State).

Thomas Bieske, University of South Florida (Session 2A)

A subelliptic fundamental solution

Abstract: In this talk, we review the history of fundamental solutions to the p-Laplace equation in sub-Riemannian spaces. We then provide a new fundamental solution outside the classes of previous known examples. This is joint work with Robert Freeman.

Dat Cao, Texas Tech University (Session 1C)

Asymptotic expansions for solutions of Navier-Stokes equations

Abstract: We study the large time behavior of solutions to Navier-Stokes equations with periodic boundary conditions in 3D. The body forces are assumed to decay algebraically. The asymptotic expansions of Foias-Saut-type for all Leray-Hopf weak solutions are investigated. We show that, if the force has an asymptotic expansion, as time goes to infinity, in terms of negative-power functions in Gevrey spaces, then any weak solution admits an asymptotic expansion of the same type.

This is a joint work with Luan Hoang (Texas Tech University).

Alan Chang, University of Chicago (Session 2C)

Analytic capacity and projections

Abstract: In the 1960s, Vitushkin conjectured that a compact set in the plane is non-removable for bounded analytic functions (or equivalently, has positive analytic capacity) if and only it has positive Favard length, or in other words, its orthogonal projections have positive length in a set of directions of positive measure. In 1986, Mattila showed that this conjecture is false. However, it is not known yet if one of the implications in Vitushkin's conjecture holds. Namely, does positive Favard length imply positive analytic capacity? We will present a result related to this

open question: if one strengthens the assumption of positive Favard length in a suitable way, then the answer is positive. This is joint work with Xavier Tolsa.

Manki Cho, Rochester Institute of Technology (Session 3A)

Steklov eigenproblems and representations of harmonic functions

Abstract: This talk will describe Steklov representations of solutions of Laplacian boundary value problems. The spectral theory of trace spaces of harmonic functions leads advantages in treating orthogonal bases of the class of all finite energy harmonic functions consisting of harmonic Steklov eigenfunctions. Laplace equations with non homogeneous boundary data will be considered and an idea to approximate harmonic functions will be discussed.

John Dever, Georgia Institute of Technology (Session 2B)

Local space and time scaling exponents for diffusion on a compact metric space

Abstract: In this talk we define and investigate a local space scaling exponent α and a local time scaling exponent β. The exponent α is the local Hausdorff dimension. We provide several examples of spaces with continuously variable α. We also investigate a local Hausdorff measure and establish uniqueness up to a strong equivalence of measures satisfying the Ahlfors regularity property $\mu(B_r(x)) \asymp r^{\alpha(x)}$. The exponent β is roughly defined as a critical exponent of the expected number of steps needed for a discrete time random walk on an approximation of the space at scale ϵ to leave a ball multiplied by a power ϵ^{β} as the scale ϵ goes to 0. This exponent can then be localized. Next, we use β to re-normalize the time scale by introducing local exponential waiting times with mean at site x of $\epsilon^{\beta(x)}$. This gives a continuous time walk. We then examine the exit time regularity condition that if $\mathcal{T}(B)$ is the supremum of the exit times from a ball B then $\mathcal{T}(B_r(x)) \asymp r^{\beta(x)}$. We show that this condition has a number of useful consequences, one of which is a Faber-Krahn type inequality $\lambda_{1,\epsilon}(B) \geq \frac{c}{R^{\beta(x_0)}}$ where $B = B_R(x_0)$, $\lambda_{1,\epsilon}(B)$ is the bottom of the spectrum of the generator of the time-renormalized walk killed outside of B at stage ϵ and c is a constant independent of ϵ, x_0, and R. Lastly, we investigate convergence of approximating forms using $\Gamma-$convergence techniques.

Zijian Diao, Ohio University (Session 1D)

Modern analysis on e and $\zeta(2)$

Abstract: The irrationality of e and evaluation of zeta(2) are two classical analysis problems which shared the marks of both Euler and Fourier. The former was proven by Euler using continued fraction and by Fourier using standard series analysis. The latter is a hallmark achievement of Euler, which also allows for a wealth of different approaches originated from multiple branches of mathematics. While Fourier never supplied his own solution to zeta(2), quantum Fourier transform, an essential tool in quantum computing, has found its way into this problem. In this talk, we will present a no-calculus-required proof of the irrationality of e and a new approach to zeta(2) as a natural consequence of the quantum counting algorithm.

Dong Dong, University of Illinois Urbana-Champaign (Session 3B)

Polynomial Roth theorem in finite fields

Abstract: I will show how harmonic analysis can be used to study the problem on the existence of certain patterns in subsets of finite fields.

Katie Elliott, Baylor University (Session 1D)

A self-adjoint operator generated by the Krall differential equation in an extended Hilbert space

Abstract: We construct a self-adjoint operator generated by the Krall differential equation in the extended Hilbert space $L^2(-1,1)$ direct summed with two copies of the complex plane. The theory we use to create this self-adjoint operator, called the GKN-EM Theorem, was developed by Littlejohn and Wellman as an application of the general Glazman-Krein-Naimark (GKN) Theorem. We also use properties of functions in the maximal domain for the differential operator generated by the Krall differential expression proved by Loveland. Through our construction of this self-adjoint operator, we find the maximal and minimal operators in the extended Hilbert space. Continuity, as a boundary condition, is forced by our construction of this self-adjoint operator. Finally, we also show that the Krall polynomials are eigenfunctions of the constructed self-adjoint operator in the extended space.

Robert Freeman, University of South Florida (Session 2A)

The $p(x)-$Laplacian in Carnot groups

Abstract: We establish the equivalence of weak and viscosity solutions to the p(x)-Laplacian in Carnot groups, under certain reasonable restrictions on the function p(x). As a consequence, we obtain a comparison principle for viscosity solutions. This then implies that viscosity solutions to Dirichlet problems are unique.

Derek Jung, University of Illinois Urbana-Champaign (Session 2A)

BiLipschitz embeddings into jet space Carnot groups not admitting Lipschitz extensions

Abstract: For all k and n, we construct a biLipschitz embedding of the n-dimensional sphere into the jet space Carnot group $J^k(R^n)$ that does not admit a Lipschitz extension to the ball. The construction is given by viewing the sphere as two glued copies of a ball and then taking the jet of a suitable smooth function. The hope is that the existence of this embedding implies the non-density of Lipschitz mappings in certain Sobolev spaces.

Haojian Li, University of Illinois Urbana-Champaign (Session 3C)

Decay to bar states

Abstract: Stable states are crucial for us to learn the partial differential equations, which has been learned for a long time. However, for 2-D Navier-Stokes equations with periodic boundary conditions and small viscosity, metastable states are long-time governing dynamics which decay slowly. Those states have been learned numerically and analytically for a long time. In this talk, a new scheme would be introduced to learn the fast decay to bar states, which is a type of metastable states. The scheme includes Bootstrap method and eigenvalue approximation. Only numerical results are given for the eigenvalue approximation.

Jiaqi Liu, University of Toronto (Session 2D)

Inverse scattering for the derivative NLS Equation with arbitrary spectral singularity

Abstract: We show that the derivative nonlinear Schrodinger equation(DNLS) in 1+1 dimension is globally well-posed in certain weighted Sobolev space. Our result exploits the complete integrability of DNLS together with Zhou's method for inverse scattering with arbitrary spectral singularities.

George Lytle, University of Kentucky (Session 1A)

Solving Calderón's Problem with rough conductivities

Abstract: Calderón's problem is to reconstruct the conductivity σ of a bounded conducting body $\Omega \subset \mathbb{R}^2$ from boundary measurements of applied voltage and induced current. There is a reconstruction algorithm due to Adrian Nachman, for $\sigma \in W^{2,p}(\Omega)$ for some $p > 1$, involving the scattering transform $\mathbf{t}$. The algorithm has been implemented numerically, and works well even for *discontinuous* conductivities provided the scattering transform is low-pass filtered. However, there is currently no theoretical framework to justify this approximation or to obtain meaningful convergence estimates. The key missing ingredient is an understanding of the scattering transform for non-compactly supported conductivities.

In this talk we will discuss preliminary results for this scattering transform. In particular, we prove existence and uniqueness for the so-called complex geometric optics solutions to the

Beltrami equation

$$\overline{\partial} f = \mu \overline{\partial f}, \quad \mu = \frac{1-\sigma}{1+\sigma}$$

when $\mu \in L^q(\mathbb{C})$ for all $1 < q < \infty$ and $\|\mu\|_\infty \leq \kappa < 1$. These solutions define the scattering transform of μ through their spatial asymptotic behavior. We'll also explain our program for obtaining convergence estimates that justify the low-pass approximation.

This is joint work with Peter Perry and Samuli Siltanen.

Jacob Makaya, Texas A&M International (Session 3C)

Static bubbles in a channel

Abstract: Consider the 2-dimension selection-problem of static bubbles in a horizontal channel. The channel is filled with incompressible viscous fluid. In one end of the channel, fluid of different viscosity is pumped. The less viscous fluid drives the more viscous one creating bubbles. For a finite number of bubbles, we use conformal mappings and reduction to symmetry to derive an explicit solution using Green's Function and Harmonic Measure.

Henok Mawi, Howard University (Session 1A)

On the numerical solution to an inverse problem of refraction

Abstract: The far field refractor problem in geometric optics is an inverse problem which deals with constructing a refracting surface that is capable of reshaping a light beam from a one point source with a given illumination intensity into a prescribed intensity distribution. In this talk we discuss an iterative scheme to solve this problem. In particular, we will describe a numerical algorithm which was first used in the work of Caffarelli, Kochengin and Oliker, in relation to synthesis of reflector surfaces and show that a simplified version of this algorithm can be extended to obtain an approximate solution for the refractor problem with arbitrary precision. We further exhibit Lipschitz regularity that will lead to the convergence in finite steps of the method when the distribution density functions are bounded and satisfy mass balance conditions.

Isaac Michael, Baylor University (Session 3B)

Birman-Hardy-Rellich-type inequalities and refinements

Abstract: In 1961, Birman proved a sequence of inequalities for smooth functions of compact support on the open half-line, containing as special cases the classical Hardy and Rellich inequalities. In this talk, we show this sequence of inequalities holds on a more general Hilbert space of functions defined on the closed half-line. We also show that Birmans inequalities are closely related to a sequence of generalized, continuous Cesro averaging operators whose spectral properties we determine.

Finally, we hint at generalizations in the multi-dimensional setting, focusing on improved inequalities with logarithmic and radial refinement terms. In particular, we derive Hardy-type multidimensional inequalities with logarithmic refinement terms, and the gradient replaced by its radial counterpart.

This is based on joint work with F. Gesztesy, L. Littlejohn, M. Pang, and R. Wellman.

Oleksandr Misiats, Virginia Commonwealth University (Session 2B)

Linear and nonlinear elastic model of shape memory alloys

Abstract: We consider the minimization problem related to modeling materials with memory, e.g. shape memory alloys. I will start my presentation with a visual illustration of shape memory effect. Physical experiments suggest that if two distinct phases of such material are present at opposite sides of a rectangular sample, a zig-zag pattern is formed. Our goal is to understand if this pattern is energy minimizing. In my talk I will describe a number of results on the energy scaling laws of both linear and nonlinear elastic models of shape memory alloys, which involve the zig-zag type transitions. In particular, in the nonlinear elastic setting we observe that the energy of the zig-zag wall decreases significantly if the stiff Dirichlet boundary condition is replaced with a slightly relaxed "semi-stiff" condition.

Iurii Posukhovskyi, University of Kansas (Session 2D)

On the normalized ground states for the Kawahara equation and a fourth order NLS

Abstract: We consider the Kawahara model and two fourth order semi-linear Schrödinger equations in any spatial dimension. We construct the corresponding normalized ground states, which we rigorously show to be spectrally stable. For the Kawahara model, our results provide a significant extension in parameter space of the current rigorous results. At the same time, we verify and clarify recent numerical simulations of the stability of these solitons. For the fourth order NLS models, we improve upon recent results on stability of very special, explicit solutions in the one dimensional case. Our multidimensional results for fourth order NLS seem to be the first of its kind. Of particular interest is a new paradigm that we discover herein. Namely, all else being equal, the form of the second order derivatives (mixed second derivatives vs. pure Laplacian) has implications on the range of existence and stability of the normalized waves.

Abba Ramadan, University of Kansas (Session 3A)

Existence of traveling waves for a class of nonlocal nonlinear equations with bell shaped kernels

Abstract: In this article we are concerned with the existence of traveling wave solutions of a general class of nonlocal wave equations: $u_{tt} - a^2 u_{xx} = (\beta * u^p)_{xx}, p > 1$. Members of the

class arise as mathematical models for the propagation of waves in a wide variety of situations. We assume that the kernel β is a bell-shaped function satisfying some mild differentiability and growth conditions. Taking advantage of growth properties of bell-shaped functions, we give a simple proof for the existence of bell-shaped traveling wave solutions.

Matt Reynolds, University of Louisville (Session 3D)

Interpolation of convolution operators on weighted Lorentz spaces

Abstract: In this talk we will introduce and discuss interpolation of operators between Banach spaces and apply it to convolution operators on weighted Lorentz spaces.

Mariana Smit Vega Garcia, University of Washington (Session 2C)

Higher regularity of the free boundary in the parabolic Signorini problem

Abstract: We show that the quotient of two caloric functions which vanish on a portion of an H^{k+a} regular slit is H^{k+a} at the slit, for $k \geq 2$. As an application, we show that the free boundary near a regular point of the parabolic thin obstacle problem with zero obstacle is C^∞ regular in space and time. This is joint work with A. Banerjee and A. Zeller.

Gareth Speight, University of Cincinnati (Session 2C)

Differentiability of Lipschitz functions in small sets

Rademacher's theorem asserts that every Lipschitz function from $\mathbb{R}^n$ to $\mathbb{R}^m$ is differentiable except on a set of measure zero. Depending on the dimensions of the spaces involved, this theorem may or may not admit a converse. If $n > 1$ then there exists a measure zero set N in $\mathbb{R}^n$ containing a point of differentiability for every Lipschitz function from $\mathbb{R}^n$ to $\mathbb{R}$. Such sets are called universal differentiability sets. We discuss universal differentiability sets in Euclidean spaces and Carnot groups. Carnot groups are spaces with much of the Euclidean structure (translations, dilations, Haar measure and a path distance) but where the shortest distance between points is usually not along a straight line.

Justin Taylor, Murray State University (Session 1B)

Eigenvalue convergence for the mixed problem

Abstract: We consider eigenvalues of an elliptic operator

$$Lu = -\frac{\partial}{\partial x_j}\left(A_{ij}\frac{\partial u}{\partial x_i}\right)$$

where $u = (u^1, ..., u^m)^T$ is a vector valued function and the coefficients A_{ij} are $m \times m$ matrices whose elements $a_{ij}^{\alpha\beta}$ are bounded and symmetric. We perturb our domain Ω_0 by adding a set of small measure, T_ε to form the domain Ω_ε. We prescribe mixed boundary conditions on quite general decompositions of the boundary and look at the behavior of the eigenvalues of Ω_ε as T_ε shrinks to zero. We look at systems which satisfy either a strong ellipticity condition, a Legendre-Hadamard condition, or in particular, the system of linear elasticity.

Hagop Tossounian, Baylor University (Session 2B)

Kac's model and approach to equilibrium using the Gabetta-Toscani-Wennberg metric

Abstract: In his quest to study the Boltzmann equation, Mark Kac invented a stochastic N-particle model and, taking marginals of the distributions and using a device known as propagation of chaos, he arrived at a Boltzmann-like equation. This model was revisited in the 90s and the L^2 spectral gap was computed explicitly. This gap is bounded below uniformly in of N as conjectured by Kac. In this talk I will introduce the Gabetta-Toscani-Wennberg metric, compare it to the L^2 metric and show a slow initial approach to equilibrium for time of order 1.

Ignacio Uriarte-Tuero, Michigan State University (Session 3B)

Two weight norm inequalities for singular and fractional integral operators in R^n

Abstract: I will report on recent advances on the topic, related to proofs of T1 type theorems in the two weight setting for Calderón-Zygmund singular and fractional integral operators, with side conditions, and related counterexamples. Joint work with Eric Sawyer and Chun-Yen Shen.

Taige Wang, University of Cincinnati (Session 3C)

Analysis on shearing flows of a thixotropic model

Abstract: The PEC (partially extending strand convection) model of Larson is able to describe thixotropic yield stress behavior in the limit where the relaxation time is large relative to the retardation time. The small parameter ε measures the ratio of retardation time to relaxation time. In this talk, we discuss the effect of slow oscillatory shearing with frequencies $\omega = \varepsilon p$, $p \in [0, 1]$. Different regimes and dynamics are developed from the transitions of phases. Moreover, the development of shear bands in planar Poiseuille flow which is started up from rest is studied. We determine the position and width of shear bands as a function of time. We identify an initial

phase of "fast yielding" during which the width of the transition between high and low shear rate regions behaves like t^{-3}. This continues until t (measured on the scale of the retardation time) is on the order of $\varepsilon^{-1/3}$. Then there is a phase of "delayed yielding" during which the width of the transition is of order "ε". Eventually, the width sharpens as $1/(\varepsilon^2 t^3)$. We also show how these results are modified by the Korteweg stresses which prevent the transition from becoming infinitely sharp and also change the location where the transition takes place.

Xin Yang, University of Cincinnati (Session 1C)

Lifespan estimate for the partial nonlinear radiation problems

Abstract: This talk is about the lifespan estimate for the heat equation $u_t = \Delta u$ in a bounded domain Ω in $\mathbb{R}^n (n \geq 2)$ with positive initial data u_0 and partial nonlinear radiation boundary conditions. First, the local existence and uniqueness of the classical solution will be discussed. Secondly, both upper and lower bounds of the lifespan will be shown. Finally, the asymptotic behaviour of the bounds concerning the nonlinearity power q, the initial data u_0 and the area of the boundary part where the nonlinear radiation occurs will be explored. This is a joint work with Zhengfang Zhou.

Shiwen Zhang, Michigan State University (Session 1B)

Refined LDT for quasiperiodic lattice Schrödinger operator and optimal Hölder continuity of the Lyapunov exponents

Abstract: We consider one-dimensional discrete quasiperiodic Schrodinger operators. Assume the potential is real analytic and assume positive Lyapunov exponents. We prove some refined Large Deviation Theorem(LDT) for any irrational frequency in an exponential regime with respect to the Lyapunov exponent. The refined LDT implies some optimal Holder continuity of the Lyapunov exponents and the integrated density of states. The Holder continuity is optimal for Liouville frequency when the Lyapunov is small. In the large coupling regime, we show that the local Holder exponent is independent of the coupling constant.

Weiqun Zhang, Wright State University (Session 3A)

A uniformly convergent numerical method for quasilinear singular perturbation problems using weak formulation

Abstract: A numerical method is proposed to solve quasilinear singular perturbation problems using weak formulation. The uniform convergence, which is independent of the singular perturbation parameter, is numerically verified.