ORAM 10
List of Abstracts

1. Plenary Speakers

Mario Bonk, University of Connecticut

Quasiregular maps and non-linear potential theory

Abstract: Non-linear potential theory is closely related to the theory of quasiconformal and quasiregular maps in higher dimensions. For example, one can use potential-theoretic methods to prove Liouville's theorem for 1-quasiconformal or the Rickman-Picard theorem for quasiregular mappings. In my talk I will outline some of the basic ideas and discuss recent developments.

Katya Krupchyk, University of California Irvine

Inverse boundary problems for elliptic PDE

Abstract: We shall provide a general introduction to the field of inverse boundary problems for elliptic PDE, with the celebrated Calderon problem serving as a prototypical example. We shall also survey some of the more recent developments, including partial data inverse problems, inverse boundary problems on Riemannian manifolds, as well as inverse boundary problems for non-linear equations. In particular, we shall see that the presence of a nonlinearity may actually help, allowing one to solve inverse problems in situations where the corresponding linear counterpart is open. This talk is based on joint works with Yavar Kian, Tony Liimatainen, Mikko Salo, and Gunther Uhlmann.

Jason Metcalfe, University of North Carolina Chapel Hill

Local energy in the presence of degenerate trapping

Abstract: Trapping is a known obstruction to local energy estimates for the wave equation and local smoothing estimates for the Schrdinger equation. When this trapping is sufficiently unstable, it is known that estimates with a logarithmic loss can be obtained. On the other hand, for very stable trapping, it is known that all but a logarithmic amount of local energy decay is lost. Until somewhat recently, explicit examples of scenarios where an algebraic loss (of regularity) was both necessary and sufficient for local energy decay had not be constructed. We will review what is known in these specific examples. We will also examine the relationship between the trapping and the existence of a boundary. In this highly symmetric case, a relatively simple proof showing a bifurcation in the behavior of local energy as the boundary passes through the trapping is available. This is related, e.g., to the instability of ultracompact neutrino stars.

Nataša Pavlović, University of Texas Austin

Beyond binary interactions of particles

Abstract: In this talk we shall discuss dynamics of systems of particles that allow interactions beyond binary, and their behavior as the number of particles goes to infinity. In particular, an example of such a system of bosons leads to a quintic nonlinear Schrodinger equation, which we rigorously derived in a joint work with Thomas Chen. An example of a system of classical particles allowing instantaneous ternary interactions leads to a new kinetic equation that can be understood as a step towards modeling a dense gas in non-equilibrium. We call this equation a ternary Boltzmann equation and we rigorously derive it in a recent work with Ioakeim Ampatzoglou. Time permitting, we will also discuss the recent work with Ampatzoglou on a derivation of a binary-ternary Boltzmann equation describing the kinetic properties of a dense hard-spheres gas, where particles undergo either binary or ternary instantaneous interactions, while preserving momentum and energy. An important challenge we overcome in deriving this equation is related to providing a mathematical framework that allows us to detect both binary and ternary interactions. Furthermore, this work introduces new algebraic and geometric techniques in order to eventually decouple binary and ternary interactions and understand the way they could succeed one another in time.

---

2. Contributed Speakers

Michel Alexis, University of Wisconsin Madison (Session 3D)

The Steklov problem for the Orthogonal Polynomials on the Unit Circle, orthogonal with respect to a Muckenhoupt weight

Abstract: Let $\{\varphi_n\}_n$ be the sequence of degree n polynomials on $\mathbb{T}$, orthonormal with respect to a positive weight w. Steklov conjectured whenever $w \geq \delta > 0$ a.e. then $\{\varphi_n\}_n$ are uniformly bounded in L^∞. While false, this conjecture brings us to ask the following: under what regularity conditions on w are $\{\varphi_n\}_n$uniformly bounded in $L^p(w)$ for some $p > 2$? We discuss some answers to this question using the contraction principle and operator estimates for the Hilbert transform, in particular recent joint work with Alexander Aptakarev and Sergey Denisov for when w is a Muckenhoupt weight.

Geoffrey Bentsen, University of Wisconsin Madison (Session 2A)

Averages over Curves in the Heisenberg Group

Abstract: In this talk I will investigate the L^p regularity of generalized Radon transforms through several model examples, culminating in a sharp result for averaging operators over certain families of curves invariant under translation in the Heisenberg group. This noncommutative setting involves degeneracies not seen in the Euclidean case. We prove L^p regularity in the "worst" case of these degeneracies, a fold blowdown singularity, using almost orthogonality arguments and iterated decoupling on the cone.

Deniz Bilman, University of Cincinnati (Session 1D)

Far-Field Asymptotics of High-Order Rogue Waves

Abstract: We give a rigorous asymptotical description of the wave field surrounding the fundamental rogue wave solutions of the nonlinear Schrödinger equation in the limit of large order, away from the peak. This is joint work with Peter D. Miller.

Chandan Biswas, Indian Institute of Science (Session 1A)

Sharp Fourier restriction estimates onto curves

Abstract: We will discuss the existence of extremizers for the Lebesgue space bounds of the Fourier restriction onto curves, particularly for the model case, the moment curve in $\mathbb{R}^d$. This is based on our joint work with Betsy Stovall.

Marco Capolli, University of Trento (Session 2A)

A C^m Lusin Approximation Theorem in the Heisenberg Group

Abstract: In the Euclidean setting, a measurable function can be made continuous after a perturbation in a set of small measure. Concerning higher regularities it is possible to give conditions under which a function can be approximated by a function of class C^m. This results relies on the classical Whitney extension theorem. In this talk I will present joint work with Andrea Pinamonti and Gareth Speight in which we studied an analogous result in the Heisenberg group.

Jacob Denson, University of Wisconsin Madison (Session 2C)

Salem Sets Avoiding Nonlinear Patterns

Abstract: If a set has large Hausdorff dimension, and also supports a measure whose Fourier transform decays at a fast rate, does it necessarily contain a family of points forming a particular patterns, such as three vertices forming an isosceles triangle? This talk will discuss new construction techniques to find sets with large dimension of the form above avoiding a particular family of patterns. The proof of these techniques involves some combinatorial estimates, and an understanding of concentration of measure phenomena.

Joshua Flynn, University of Connecticut (Session 1C)

Sharp Caffarelli-Kohn-Nirenberg Inequalities for Grushin Vector Fields and Iwasawa Groups.

Abstract: Caffarelli-Kohn-Nirenberg inequalities are established for the Grushin vector fields and for Iwasawa groups (i.e., the boundary group of a real rank one noncompact symmetric space). For all but one parameter case, this is done by introducing a generalized Kelvin transform which is shown to be an isometry of certain weighted Sobolev spaces. For the exceptional parameter case, the best constant is found for the Grushin vector fields by introducing Grushin cylindrical coordinates and studying the transformed Euler-Lagrange equation.

Landon Gauthier, University of Kentucky (Session 1B)

A Perturbed Polyharmonic Inverse Problem

Abstract: The goal is to see how much we can perturb the polyharmonic operator using first and zero order terms. This is done by constructing normed spaces that play well with our operator and computing an approximate right inverse to construct CGO solutions. Then using norm estimates on our solution and the perturbed potential to recover our potentials.

Wesley Hamilton, University of North Carolina Chapel Hill (Session 2B)

Consistency of spectral flows on graphs

Abstract: Consistency problems consider the limiting behavior of graphs constructed from point clouds sampled from an underlying compact manifold, and the convergence of graph operators (like the graph Laplacian) to the corresponding continuum operators (like the Laplace-Beltrami operator). In this talk I?ll survey the consistency framework, and then discuss recent work on the consistency of Dirichlet-to-Neumann operators on graphs with boundary. The primary application of this is establishing consistency of a family of spectral flows used for counting the nodal deficiencies of Laplace eigenvectors/eigenfunctions.

Sam Herschenfeld, University of Kentucky (Session 2B)

A Simplified Proof of a continuum Minami Estimate

Abstract: Random Schrödinger operators, $H_\omega = H_0 + V_\omega$ model a particle in a disordered material. One way to express the long conjectured metal-insulator transition is through local eigenvalue statistics. Minami estimates bound the probability that H_ω restricted to a finite box has more than one eigenvalue in a small interval and are key to this field. Minami estimates have been around for awhile in the lattice. Klopp proved a Minami estimate in a 1D continuum model in 2014 and Dietlein and Elgart proved one for any dimension in an interval at the bottom of the spectrum in 2018.

We'll discuss Klopp's strategy as an intuitive approach attempting to avoid lattice techniques and give a simplified proof that avoids some technical sections.

Ramesh Karki, Indiana University East (Session 2D)

An approach for recovering initial temperature via bounded linear time sampling

Abstract: We construct finitely many time instances with a linear growth in a given bounded interval such that the temperature measurements of a thin uniform one-dimensional rod taken at a fixed point of the rod and at these finite time instances lead to the recovery of the initial temperature profile with a desired accuracy.

Blake Keeler, University of North Carolina Chapel Hill (Session 2B)

Two-Point Weyl Laws and Monochromatic Random Waves

Abstract: In this talk, I will discuss the two-point Weyl law on a compact manifold M, with the goal of understanding the statistical properties of monochromatic random waves. These waves can be thought of as randomized approximate eigenfunctions, the statistics of which are determined by their two-point correlation functions. The correlation function coincides exactly with a rescaled version of the spectral function of the Laplace-Beltrami operator, so the two-point Weyl law corresponds directly with the high-frequency behavior of monochromatic random waves. I will discuss my result which shows that in the setting of manifolds without conjugate points, one can improve the remainder in the two-point Weyl law by a logarithmic factor, and that this improvement is preserved when differentiating the spectral function. An important corollary of this theorem is that the correlation function of a monochromatic random wave has a local limit which is independent of the geometry and topology of M as the frequency parameter tends to infinity. Furthermore, one has an explicit inverse logarithmic rate of this convergence in the C^∞-topology.

Shi-Zhuo Looi, University of Kentucky (Session 1D)

Pointwise decay for the energy-critical nonlinear wave equation on perturbations of Minkowski spacetime

Abstract: On Minkowski spacetime, the optimal decay rate for the energy-critical nonlinear wave equation with smooth and compactly supported initial data is $\langle t\rangle^{-1}\langle t-r\rangle^{-3}$ and was shown by Grillakis. In this talk, we outline the ideas used in proving the optimal decay rate everywhere for this problem with a potential on perturbations of the Minkowski spacetime that decay at spatial infinity. This decay rate depends on how rapidly the vector fields of the metric coefficients and of the potential decay at spatial infinity and the method of proof recovers Grillakis? result on the Minkowski spacetime. The main ideas used include local energy decay and Strichartz estimates for vector fields of the solution.

Joseph Miller, University of Texas Austin (Session 2D)

Derivation of Boltzmann's Equation for Mixtures

Abstract: In joint work with Natasa Pavlovic and Ioakeim Ampatzoglou, we rigorously derive a Boltzmann equation for mixtures from the dynamics of two types of hard sphere gases. We prove that the microscopic dynamics of two gases with different masses and diameters are well defined, and introduce the concept of a two parameter BBGKY hierarchy to handle the non-symmetric interaction of these gases. As a corollary of the derivation, we prove Boltzmann's propagation of chaos assumption for the case of a mixtures of gases.

Oleksandr Misiats, Virginia Commonwealth University (Session 3A)

Singular perturbation of an elastic energy with a singular weight

Abstract: In my talk I will describe the singular perturbation of an elastic energy with a singular weight, which naturally emerges in modelling shape memory alloys, as well as diblock copolymers. The minimization of this energy results in a multi-scale pattern formation. I will start with addressing the derivation of an energy scaling law in terms of the perturbation parameter. Next, although one cannot expect periodicity of minimizers, I will illustrate that the minimal energy is uniformly distributed across the sample. Finally, following the approach developed by Alberti and Muller, we proved that a sequence of minimizers of the perturbed energy converges to a Young measure supported on piecewise-linear periodic functions. This is a joint project with Ihsan Topaloglu (VCU) and Daniel Vasiliu (William and Mary).

Evangelos Nastas, SUNY Albany (Session 2C)

An inequality bound improvement and some proofs related to a theorem of Malliavin

Abstract: This talk is devoted to an inequality bound improvement that?s an estimated inversion formula related to the Cauchy integral of a distribution function on $\mathbb{R}_+$. It yields a theorem proven by Malliavin. Improvement and so alternatives to the inequality are presented. The inequality has been applied in spectral theory of differential and pseudo-differential operators and by extension to mathematical physics, notably quantum mechanics, e.g. atomic spectra, and more generally physics of vibrations and so is expected for its improved version and associated theorems.

Adam Osekowski, University of Warsaw (Session 2C)

Fefferman's inequality and its offspring

Abstract: A celebrated result of Fefferman asserts that the dual of the Hardy space H^1 is the class BMO of functions of bounded mean oscillation. We will discuss a quantitative version of this result and identify the best constant in the corresponding estimate. We will also present a number of related inequalities arising in the weighted theory and probability.

Woongbae Park, Michigan State University (Session 2D)

Bubbling and Neck anaylsis of harmonic maps

Abstract: The harmonic map equation is a conformally-invariant nonlinear elliptic equation that exhibits a well-known concentration phenomenon: sequences of solutions concentrate at points and can be renormalized to produce a "bubble tree" solution as a limit. However, the behavior of the sequence in the neck regions between bubbles is not understood. This talk describes new results on the energy and length of these necks.

Maximilian Pechmann, University of Tennessee Knoxville (Session 2B)

On a condition for type-I Bose-Einstein condensation in random potentials in d dimensions

Abstract: We discuss Bose-Einstein condensation (BEC) in systems of pairwise non-interacting bosons in random potentials in d dimensions. Working in a rather general framework, we show a ?gap condition? which is sufficient to conclude existence of type-I BEC in probability and in the rth mean. We illustrate our results in the context of the well-known (one-dimensional) Luttinger-Sy model. Here, whenever the particle density exceeds a critical value, we show in addition that only the ground state is macroscopically occupied. This is joint work with Joachim Kerner and Wolfgang Spitzer.

Mohamed Mahabubur Rahman, Texas Tech University (Session 2D)

Regularity criteria for the two and three dimensional Kuramoto-Sivashinsky equation

Abstract: The regularity criteria for the 2D and 3D Kuramoto-Sivashinsky equation is discussed in both its scalar and vector forms. In particular, we examine integrability criteria for the regularity of solutions in terms of the scalar solution ϕ, the vector solution $u := \nabla\phi$,as well as the divergence $\mathrm{div}(u) = \Delta\phi$, and each component of u and ∇u. This is a joint work with Prof. Adam Larios and Prof. Kazuo Yamazaki.

Antoine Remond-Tiedrez, University of Wisconsin Madison (Session 3A)

Instability of an Anisotropic Micropolar Fluid

Abstract: Many aerosols and suspensions, or more broadly fluids containing a non-trivial structure at a microscopic scale, can be described by the theory of micropolar fluids. The resulting equations couple the Navier-Stokes equations which describe the macroscopic motion of the fluid to evolution equations for the angular momentum and the moment of inertia associated with the microscopic structure. In this talk we will discuss the case of viscous incompressible three-dimensional micropolar fluids. We will discuss how, when subject to a fixed torque acting at the microscopic scale, the nonlinear stability of the unique equilibrium of this system depends on the shape of the microstructure.

Jacob Shapiro, University of Dayton (Session 3D)

Semiclassical resolvent estimates for long range Lipschitz potentials

Abstract: We give an elementary proof of weighted resolvent estimates for the semiclassical Schrödinger operator $-h^2\Delta + V(x) - E$ in dimension $n \neq 2$, where h, $E > 0$. The potential is real-valued and V and $\partial_r V$ exhibit long range decay. The resolvent norm grows exponentially in h^{-1}, but near infinity it grows linearly. When V is compactly supported, we obtain linear growth if the resolvent is multiplied by weights supported outside a ball of radius $CE^{-1/2}$ for some $C > 0$. This E-dependence is sharp and answers a question of Datchev and Jin. This talk is based on joint work with Jeffrey Galkowski.

Jonathan Stanfill, Baylor University (Session 1C)

A refinement of Hardy's inequality

Abstract: We prove the inequality

$$\int_0^\pi dx\,|f'(x)|^2 \geq \frac{1}{4}\int_0^\pi dx\,\frac{|f(x)|^2}{\sin^2(x)} + \frac{1}{4}\int_0^\pi dx\,|f(x)|^2, \quad f \in H_0^1((0,\pi)),$$

where both constants $1/4$ appearing in the above inequality are optimal and the inequality is strict in the sense that equality holds if and only if $f \equiv 0$. This inequality is derived with the help of the exactly solvable, strongly singular, Dirichlet-type Schrödinger operator associated with the differential expression

$$\tau_s = -\frac{d^2}{dx^2} + \frac{s^2 - (1/4)}{\sin^2(x)}, \quad s \in [0,\infty),\ x \in (0,\pi).$$

This new inequality represents a refinement of Hardy's classical inequality

$$\int_0^\pi dx\,|f'(x)|^2 \geq \frac{1}{4}\int_0^\pi dx\,\frac{|f(x)|^2}{x^2}, \quad f \in H_0^1((0,\pi)),$$

and one of its well-known extensions. In addition, we hint at the possibility to extend this inequality to more general situations where the differential expression is of the form $\tau = -(d^2/dx^2)+q(x)$, $x \in (a,b) \subset \mathbb{R}$, where $q(\,\cdot\,)$ behaves like $C_c(x-c)^{-2}$ near $x = c$, $c \in \{a,b\}$, with $C_c \geq -1/4$. This is based on joint work with Fritz Gesztesy and Michael Pang.

Alex Stokolos, Georgia Southern University (Session 3C)

On a size of univalent and typically real in the unit disc polynomials

Abstract: We would like to address the problem of estimating the quantities $\sup\max|F_n(z)|$ and $\sup\min|F_n(z)|$ where supremum is taking over all polynomials univalent or typically real in the unit disc while max and min are taken over specific subsets of the closed units disc, including the disc itself. Fundamental work in this direction is due to W.W.Rogosinski, G.Szego, C.Michel, M.Brandt and some others. In particular, Brandt solved a problem of the maximal size of univalent polynomials and showed that the extremizers are polynomials invented by Ted Suffridge. Also, he solved Michel's problem of estimating the modulus of a typically real polynomial of odd degree. In the talk we will present the complete answer to Michel's problem. These are joint results with Dmitriy Dmitrishin, Paul Hagelstein and Andrey Smorodin.

Dmitry Stolyarov, St. Petersburg State University (Session 1A)

Hardy–Littlewood–Sobolev inequality for $p = 1$

Abstract: The Hardy–Littlewood–Sobolev inequality is a useful tool in analysis. It says that the Riesz potential I_α maps $L_p(\mathbb{R}^d)$ to $L_q(\mathbb{R}^d)$ iff $1/p - 1/q = \alpha/d$ and $1 < p < q < \infty$. It appears

that if one imposes additional translation and dilation invariant assumptions on the functions in question, then the inequality becomes true in the limit case $p = 1$ (the examples are given by classical Hardy's inequalities for analytic functions, the Gagliardo–Nirenberg inequality, and more recent Bourgain–Brezis inequalities). I will state a new general theorem in this spirit.

Brandon Sweeting, University of Cincinnati (Session 2C)

On the John-Nirenburg Constant of BMO^p*,* $0 < p < 1$

Abstract: We present sharp L^p lower bounds for logarithms of A_∞ weights as a means of estimating the John-Nirenberg constant of the space $\mathrm{BMO}^p, 0 < p < 1$. The corresponding Bellman function solves the homogeneous Monge-Ampre equation, but the geometry of the solution goes beyond established theory due to the lack of regularity in the boundary condition. This is joint work with Leonid Slavin.

Vasily Vasyunin, St. Petersburg State University (Session 3B)

Sharp multiplicative inequalities with BMO

Abstract: We present the best possible constant C in the inequality

$$\|\varphi\|_{L^r} \leq C\|\varphi\|_{L^p}^{\frac{p}{r}}\|\varphi\|_{\mathrm{BMO}}^{1-\frac{p}{r}}$$

for all possible values of parameters p and r. We employ the Bellman function technique to solve this problem in the case of an interval and transfer our results to the circle and the line. Joint work with D. Stolyarov, P. Zatitskiy and I. Zlotnikov.

Vyron Vellis, University of Tennessee Knoxville (Session 3C)

Geometry of Quasiconformal trees

Abstract: A quasiconformal tree is a doubling metric tree in which the diameter of each arc is bounded above by a fixed multiple of the distance between its endpoints. In this talk, we construct a catalog of metric trees in a purely combinatorial way, and show that every quasiconformal tree is bi-Lipschitz equivalent to one of the trees in our catalog. We then discuss how such constructions apply to a special class of bounded turning and doubling metric spaces. This is inspired by results of Herron-Meyer and Rohde for quasi-arcs. The talk is based on joint work with G. C. David.

Lili Yan, University of California Irvine (Session 1B)

Inverse boundary problems for biharmonic operators in transversally anisotropic geometries

Abstract: We study inverse boundary problems for first order perturbations of the biharmonic operator on a conformally transversally anisotropic Riemannian manifold of dimension $n \geq 3$. We show that a continuous first order perturbation can be determined uniquely from the knowledge of the set of the Cauchy data on the boundary of the manifold provided that the geodesic X-ray transform on the transversal manifold is injective.

Pavel Zatitskii, St. Petersburg State University (Session 3B)

Distribution of martingales with bounded square functions

Abstract: We study the terminate distribution of a martingale whose square function is bounded. We obtain sharp estimates for the exponential moments and p-moments, as well as for the distribution function itself. The proofs are based on the elaboration of the Burkholder method and on the investigation of certain locally concave functions. The talk is based on joint work with D. Stolyarov, V. Vasyunin and I. Zlotnikov.

Scott Zimmerman, OSU Marion (Session 2A)

Singular integrals on regular, smooth curves in Carnot groups and Banach duals

Abstract: The modern study of singular integral operators on curves in the plane began in the 1970's. Since then, there has been a large volume of work done regarding the boundedness of singular integral operators defined on lower dimensional sets in Euclidean spaces. In recent years, mathematicians have attempted to push these results into a more general metric setting. In this talk, I will discuss some recent progress with V. Chousionis and S. Li regarding SIOs along $C^{1,\alpha}$ curves in Carnot groups and a similar result for curves in the dual of any separable Banach space.