1. Sam takes his favorite number, multiplies it by 3 and then adds 7 to obtain 40. What is Sam’s favorite number?

Answer: 11

2. If the sum of two numbers is 20 and the product is 96, what is the smaller number?

Answer: 8

3. If a rectangle has perimeter 20 inches and area of 24 square inches, what is the length of the shorter side?

Answer: 4

4. Gretchen is five years older than Sam and three years ago Gretchen was twice as old as Sam. How old is Sam today?

Answer: 8

5. The number $n!$ is the product $n(n-1)(n-2)\cdots1$. How many zeroes are at the end of $15!$?

Answer: 3

6. How many zeroes are at the end of $90!$?

Answer: 21

7. What is the smallest prime factor of 2010?

Answer: 2

---

1CATS stands for CATS Are Top Solvers.
8. What is the largest prime factor of 2010?

Answer: 67

9. Suppose $n$ is a positive integer. The remainder when $n$ is divided by 5 is 1 and the remainder when $n$ is divided by 7 is 2. What is the smallest possible value of $n$?

Answer: 16

10. According to the standard convention, $1 + 4/2 + 3 = 1 + 2 + 3 = 6$. Including this answer, how many different answers can you obtain by using parentheses to carry out the operations in a different order?

Answer: 4

11. Find two solutions to the equation:

$$\frac{2}{1 + \frac{2}{1 + \frac{2}{1 + x}}} = x.$$

Answer: $-2, 1$

12. Compute the product

$$\left(1 + \frac{1}{2}\right) \left(1 + \frac{1}{3}\right) \left(1 + \frac{1}{4}\right) \cdots \left(1 + \frac{1}{199}\right).$$

Answer: 100

13. The decimal expansion of $N$ is 0.4444\ldots. What is the decimal expansion of $\sqrt{N}$?

Answer: 0.6666\ldots

14. Let

$$N = 12345678910111213\ldots 9899100.$$  

What is the remainder when $N$ is divided by 9?

Answer: 1
15 In the picture, the small triangles are all equilateral and have area 1 square foot. What is the area of $\Delta ABC$?

![Triangle Diagram]

Answer: $11 \text{ ft}^2$

16 Expand $(1 + x)^{12}$ in powers of $x$:

$$(1 + x)^{12} = 1 + 12x + \cdots + x^{12}.$$ 

How many terms have even coefficients?

Answer: 9

17 Find the largest 2-digit number $A$ such that $A^2$ ends with the same two digits as $A$.

Answer: 76

18 In the picture, $ABDE$ is a square, $BCD$ is an equilateral triangle. Find the measure of $\angle DCE$.

Answer: $15^\circ$
19 Each person in the room shakes hands once with every other person, a total of 136 handshakes. How many people are there in the room?

Answer: 17

20 A rectangular piece of paper, when folded in two, has a rectangular shape similar to the original shape. If the shortest side of the piece of paper (before folding) is 8 inches, what is the longest side?

Answer: $8\sqrt{2}$

21 In the picture, the area of the large square is 50% larger than the area of the small square. Assuming that $b > a$, find the ratio $b/a$.

Answer: $2 + \sqrt{3}$

22 In the picture, $\angle ACB$ and $\angle DEB$ are right angles, $|AC| = 12, |CB| = 20$ and $|AE| = |EB|$. Find the area of the quadrilateral $ACDE$.

Answer: $\frac{396}{5}$
23 In the picture, $|AB| = |AC|$, the measure of $\angle BAD$ is $30^\circ$, and $|AE| = |AD|$. Find the measure of $\angle EDC$.

![Diagram of triangle ADE with angles labeled]

Answer: $15^\circ$

24 The expansion of $(a + b + c)^3$ is

$$(a + b + c)^3 = a^3 + b^3 + c^3 + 3ab^2 + 3ac^2 + 3bc^2 + 3a^2b + 3a^2c + 3b^2c + 6abc$$

and has 10 terms. How many terms does the expansion of $(a + b + c)^{10}$ have?

Answer: $66$

25 Let

$$f(x) = -\frac{1}{1+x}.$$  

(1) Compute $f(f(f(x)))$.

Answer: $x$

(2) Compute $f(f(f(\cdots f(1)\cdots)))$ (2000 $f$’s).

Answer: $-2$