

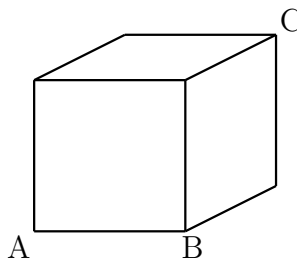
CATS COMPETITION¹
University of Kentucky High School Math Day
8 November 2008

- 1 If $3^{2x-2} = 7$, what is 9^{x-1} ?
- 2 The sides of a triangle are 4, 9, 11 inches. Find the sides of a similar triangle whose perimeter is 40 inches.
- 3 If $x = 3$ and $x = 1/6$ are solutions of the equation

$$ax^2 + bx + 1 = 0,$$

find a and b .

- 4 A car goes 51 miles per hour for 2 hours and then 66 miles per hour for 3 hours. What is the average speed of the car for the 5 hour trip?
- 5 In the cube below, $|AB| = 1$. Find $|AC|$.



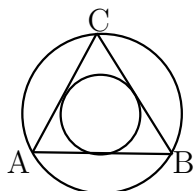
- 6 If N^2 is a divisor of $8! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8$, what is the largest possible integer value of N ?
- 7 The arithmetic mean of three numbers x, y, z is 24. The arithmetic mean of $x, 2y, z - 7$ is 35. What is the arithmetic mean of x and z ?
- 8 Cucumbers are normally 96% water. 100 pounds of cucumbers were left in the sun and are now 92% water. What is the weight of the cucumbers now?
- 9 Find the least common multiple of 77 and 21.
- 10 What is the last digit of 9^{2008} ?

¹CATS stands for CATS Are Top Solvers.

- 11 What are the last two digits of 7^{999} ?
- 12 If we make one straight cut through a pizza, we obtain two pieces of pizza. If we make another cut, we obtain three or four pieces of pizza depending on how we cut. What is the largest number of pieces we can obtain after five cuts?
- 13 Find the sum

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots + \frac{1}{199 \cdot 201}.$$

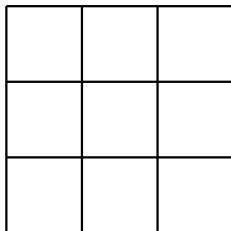
- 14 Factor $x^4 + 4$ as a product of two polynomials.
- 15 Suppose that the area of a rectangle is 20 square units and its perimeter is 20 units. The length of the diagonal is \sqrt{n} where n is a whole number. Find n .
- 16 License plates in Mathfunland use 4 digit numbers. Only the digits from 1 to 9 are used (0 not allowed), and all four digits must be different. How many different license plates are possible in Mathfunland?
- 17 If we expand the expression $(x - 3y)^2 = x^2 - 6xy + 9y^2$, the coefficients are 1, -6 and 9 and the sum of these coefficients is 4. Find the sum of the coefficients of $(x - 2y)^{10}$.
- 18 Six distinct straight lines are drawn on the plane. What is the largest possible number of intersections these lines can have?
- 19 $\triangle ABC$ is equilateral. If the area of the inscribed circle is 1, what is the area of the circumscribed circle?



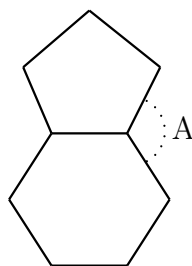
- 20 In a game, players can score either 7 or 3 points each time. If a player scores 82 points, what are the possible numbers of 3 point goals that were scored?
- 21 Find the sum of the even numbers from 2 to 2008, $2 + 4 + 6 + \cdots + 2008$.
- 22 If $n!$ stands for the product $1 \cdot 2 \cdot 3 \cdots (n - 1)n$, evaluate

$$\frac{10!}{7! 6!}.$$

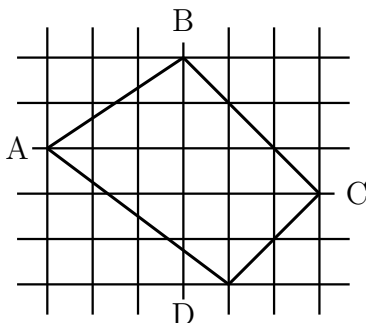
- 23 How many different squares are drawn in the following grid:



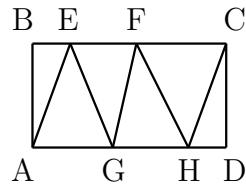
- 24 The number $n!$ is the product of the whole numbers from 1 to n . Thus $5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$. The number 120 ends with one zero. How many consecutive zeros are there at the end of $100!$?
- 25 The sketch shows a regular pentagon and a regular hexagon that share a side. Find the measure of the angle A in degrees.



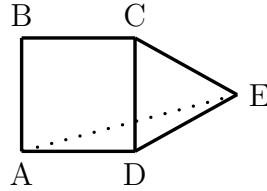
- 26 Factor $9991 = A \cdot B$, where A and B are whole numbers with $1 < A < B$. Find A .
- 27 The little squares in the grid below are 1×1 . Find the area of the quadrilateral $ABCD$.



- 28 If $n = 10101$ in base 2, give the number n in base 10.
- 29 We have a two digit prime number and when the digits are interchanged the value of the number decreases by 36. What is the number?
- 30 The area of the rectangle $ABCD$ is 6 square units. What is the combined area of the three triangles $\triangle AEG$, $\triangle GFH$, and $\triangle HCD$?



- 31** $ABCD$ is a square, the triangle $\triangle CDE$ is equilateral. Find the measure of the angle $\angle DAE$ (in degrees).



- 32** List all the integers $n \geq 0$ for which $2^n + 1$ is the square of an integer.
- 33** If $x + y = 10$ and $xy = 5$, what is $x^3 + y^3$?