**Title:** Analysis of the almost axisymmetric flow energy.

**Abstract:** The almost axisymmetric flow describes the evolution of a fluid in motion in a time dependent free boundary domain

$$D_h^t := \{(\lambda, \rho, z) \mid 0 \le \rho \le h(t, \lambda, z) \le 1, z \in [0, 1]\}.$$

The total energy can be expressed in terms of a measure $\sigma$ :

$$\mathcal{H}(\sigma) = \int_0^{2\pi} H\big(\sigma(\lambda, \cdot, \cdot)\big) d\lambda.$$

We show that despite the lack of compactness and – more seriously – the lack of convexity of the functional to be minimized below, (i) the height $h$ can be recovered from $\sigma$ as the unique minimizer of

$$H(\sigma) = \bar{H}(\sigma) + \frac{1}{2} \inf_h \Big\{ W_{\mathbb{R}^2}^2\Big(\sigma, \frac{\chi_{D_h}}{4(1-\rho)^2}\Big) + \int_{D_h} \Big(\frac{f^2}{8(1-\rho)^3} - \frac{\rho^2 + z^2}{4(1-\rho)^2}\Big) d\rho dz \Big\}.$$

(ii) The boundary of $D_h^t$ is Lipschitz and (iii) the geopotential satisfies the predicted condition

$$\varphi(t, \lambda, h(t, \lambda, z), z) = 0 \quad \text{on} \quad \partial\{h > 0\}.$$

Establishing the smoothness properties required for computing the Hamiltonian vector field $\mathbf{X}_{\mathcal{H}}$ remains a challenge (This talk is based on a joint work with M. Cullen and M. Sedjro).